Math 501-17A Exercise Sheet 2

(1) If $S = \mathbb{Q} \cap (0,1)$ show that $glb(S) = 0$ and $lub(S) = 1$.

(2) If $S \subset T$ are non-empty subsets of $\mathbb{R}$ show that $lub(S) \leq lub(T)$.

(3) If $a_n := 1 + \frac{1}{2} + \dots + \frac{1}{n}$ show that $(a_n)_{n \in \mathbb{N}}$ is not Cauchy by showing $a_{2n} - a_n \geq \frac{1}{2}$ $\forall n \in \mathbb{N}$. Hence prove that $\lim_{n \to \infty} a_n = \infty$.

(4) Compute the Taylor series for $\frac{x}{1+x^2}$ about $x = 0$ and find its region of convergence in $\mathbb{R}$.

(5) Find the global maxima and minima for $f(x) = 2x^2 - 3|x|$ on $[-1, 1]$

(6) Use the Taylor expansion of order 4 of $\sin(x)$ to evaluate

$$\lim_{x \to 0} \frac{\sin(x) - x + x^3}{x^3 + x^7}$$

(7) Investigate the max, min and pts of inflection of

$$f(x) = x^4 - x^5 \quad \text{and} \quad g(x) = x^5 + x + 1 \quad \text{on } \mathbb{R}.$$

(8) Expand $f(x) = x^3 - x^4$ in a Taylor series with remainder of order 4 and then 5 about $x = 0$. Interpret graphically, and show how the expansions of order 2 & 3 about $x = 0$ approximate the function.