University of Waikato
Department of Mathematics
Modern Algebra math310-10A Rings and Fields Assignment 4

Due Friday 28th May 2010

Hand in your work, clearly labelled with your name, id number and the number of the assignment, through the slot "Modern Algebra/310" outside the Mathematics office.

1. Find a root of $f(x) = x^3 + x^2 - 12$ in $\mathbb{Z}$. Hint: the integer factors of 12 are $\pm 1, \pm 2...$ so all you need to do is check these as roots. Then use Vieta's method to find a root. Divide this out and solve the resulting quadratic to find the other roots and express them as complex numbers.

2. Find the minimal polynomial for $\sqrt{-5} + \sqrt{3}$ over $\mathbb{Q}$ including a demonstation that it is irreducible.

3. Show that $\mathbb{Q}(\sqrt{2}, \sqrt{3}) = \mathbb{Q}(\sqrt{2} - \sqrt{3})$

4. Find the splitting field for $x^3 - 2$ over $\mathbb{Q}$ and express your answer in the form $\mathbb{Q}(\theta)$.

5.Find an irreducible polynomial $f(x) \in \mathbb{Q}[x]$ such that $\mathbb{Q}(\sqrt{1 + \sqrt{3}})$ is isomorphic to $\mathbb{Q}[x]/\langle f(x) \rangle$.

6. Find $a, b, c \in \mathbb{Q}$ such that

$$(1 + 2^{\frac{2}{3}})/(2 - 2^{\frac{1}{3}}) = a + b2^{\frac{1}{3}} + c2^{\frac{2}{3}}.$$

Indicate how this illustrates the structure of an explicit $F[x]/\langle f(x) \rangle$.

Kevin Broughan

21st May 2010