University of Waikato
Department of Mathematics
Modern Algebra math310-10A Rings and Fields
Assignment 2

**Due Friday 14th May 2010**

Hand in your work, clearly labelled with your name, id number and the number of the assignment, through the slot "Modern Algebra/310" outside the Mathematics office.

1. If an ideal $A$ of a ring $R$ contains any unit $u$ show that $A = R$

2. Find all of the maximal ideals of the ring $\mathbb{Z}/6\mathbb{Z}$.

3. If $A$ and $B$ are ideals of a ring $R$ define the product $AB$ and prove that $AB \subset A \cup B$. If $A + B = R$ show that $AB = A \cup B$.

4. In $\mathbb{Z}$ find a simple expression for $\langle 4 \rangle \langle 6 \rangle$ and conjecture its generalization to $\langle m \rangle \langle n \rangle$.

5. If $I \subset R$ is an ideal and $J \subset R/I$ an ideal of the quotient ring, prove that $J = f(K)$ for some ideal $K \subset R$, where $f : R \to R/I$ is the natural quotient map $x \to [x]$.

6. Find all of the zeros/roots of $x^3 + 3x^2 + x + 2$ in $\mathbb{Z}/6\mathbb{Z}$ and then in $\mathbb{Z}/5\mathbb{Z}$.

7. Let $f(x) = x^3 + 2x + 4$ and $g(x) = 3x + 2$. Divide $f(x)$ by $g(x)$ in $\mathbb{Q}[x]$ determining the quotient and remainder. Then "make $g(x)$ monic" by writing $g(x) = 3(x + 2/3)$ and do the division by first dividing by $x + 2/3$ and then the result of that by 3. Compare.

8. For the same polynomials $f(x)$, $g(x)$ as in 7. do the division in $\mathbb{Z}/5Z[x]$.

Kevin Broughan
**7th May 2010**