

**The University of Waikato Department of
Mathematics
Elements of Analysis Workshop/Tutorial 4:
math252-10B**

For the Workshop/Tutorial on 16th September, attempt questions 1,2,3 and 4.

Riemann Integration:

1. Let $T \subset \mathbb{R}$ be a bounded subset and $c > 0$ a real number. Prove that $\sup c.T = c. \sup T$.

2. By using the uniform partition $(P_n : n \in \mathbb{N})$ and explicit lower and upper sums show that the function $f(x) = x + 1$ is Riemann integrable on $[0, 1]$ by verifying $U(f, P_n) - L(f, P_n) \leq \epsilon_n$ where ϵ_n is some explicit function of n which tends to 0 as $n \rightarrow \infty$. Then find the value of the integral using

$$L(f, P_n) \leq \int_0^1 f \leq U(f, P_n).$$

3. Let $f(x) = 0$ for $0 \leq x < 1$ and $1 < x \leq 2$ and $f(1) = 1$. By using the partition $P_n = (0, 1 - 1/n, 1 + 1/n, 2)$ show that f is Riemann integrable and evaluate $\int_0^2 f$. Generalize this result to a function which is non-zero at at most a finite number of points $\{a_1, \dots, a_n\}$ on an interval $[a, b]$.

4. Define $q(x) = 1$ if $x \in \mathbb{Q}$ and $g(x) = 0$ otherwise. Prove that q is not Riemann integrable on $[0, 1]$ by showing that for all partitions P ,

$$U(q, P) - L(q, P) \geq 1$$

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15th September 2010