The University of Waikato
Department of Mathematics

Elements of Analysis math252-10B Tutorial 2, 29th July 2010

1. Verify that the series $\sum_{n=1}^{\infty} \frac{2n-1}{n+1}$ diverges by applying the necessary condition for convergence, $a_n \to 0$.

2. Verify that the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ diverges by applying Abel's necessary condition for convergence: if the terms are all positive and $a_n \geq a_{n+1}$ for all n and $\sum_{n=1}^{\infty} a_n$ converges, then $na_n \to 0$. Then show this using the integral test.

3. Show that the series

$$\sum_{n=1}^{\infty} (\frac{1}{4})^n$$

converges using D'Alembert's ratio test. Then do the same for

$$\sum_{n=0}^{\infty} \frac{3^n - 1}{4^n}$$

using the limit form of D'Alembert's test, without finding the sum.

4. Assume the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. Use the comparison test to show that the series

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$$

diverges also. Then by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ show that

$$\sum_{n=0}^{\infty} \frac{n + 2^n}{n^2 2^n}$$

converges.