

The University of Waikato
Department of Mathematics

Elements of Analysis math252-09B Tutorial/Workshop 6, 16th
September 2009, and Assignment 3.

Name: _____

Hand in this sheet at the end of the Workshop. Your name will be noted, but the working will not be assessed. Use both sides of this sheet and include at least some working on it. Assignment 3 is to do questions 4,5, 6 and 7. It is due Wednesday 23rd September.

1. Let $f(x) = x(x - 1)(x - 2)$. Sketch the graph of $y = f(x)$.

2. Apply Rolle's theorem to $f(x)$ on $[0, 1]$ and then find a point $x = \xi$ such that $0 < \xi < 1$ and $f'(\xi) = 0$.

3. Apply the Mean Value theorem to $f(x)$ on $[1, 3]$ and then find all of the points $x = \xi$ in $(1, 3)$ such that $f'(\xi) = (f(3) - f(1))/(3 - 1)$.

4. Give the Taylor expansion for $f(x)$ about $x = 0$ in the form

$$f(0 + h) = f(0) + f'(0)h + \frac{1}{2!}f''(0 + \theta h)h^2$$

and then, if $h = 0.1$, find θ explicitly and verify $|\theta| < 1$.

5. For the four term plus remainder Taylor expansion for $f(x)$ about $x = a$, explain why the remainder term $f(a + \theta h)h^4/4!$ is always zero so

$$f(a + h) = f(a) + f'(a)h + \frac{1}{2!}f''(a)h^2 + \frac{1}{3!}f'''(a)h^3$$

exactly for all h, a . Then do this explicitly.

6. What is a point of inflection? Find all the points of inflection of the given $f(x)$ and the slope of the tangent to the graph of $f(x)$ at the point or points, verifying the property.

7. Let $g(x) = x^4(x - 1)(x - 2)$. Find the nature of the critical point at $x = 0$. By expanding the definition, find the Taylor series for $g(x)$ about $x = 0$ including all terms, even when they are 0.

16th September 2009