The University of Waikato Department of Mathematics

Elements of Analysis Worshop/Tutorial for 24th September and Assigment 4: math252-08B

For the Workshop/Tutorial on 24th September, attempt questions 1,2,3 and 4. For **assignment 4, due now Monday 6th October, do 1-5**.

Uniform convergence of sequences and series of functions:

1. For each $n \in \mathbb{N}$ and $x \in \mathbb{R}$ let

$$f_n(x) = \frac{x^2}{n}.$$

Show that the sequence of functions $(f_n : n \in \mathbb{N})$ converges pointwise to the zero function h_0 on $\mathbb{R}$.

2. Now show the convergence of the sequence in 1. is uniform on $[0, 1]$ and then $[0, 2]$

3. Keeping to this same sequence show the convergence is uniform on $[-c, c]$ for every real $c > 0$.

4. Now verify each f_n is continuous and deduce h_0 is continuous on $\mathbb{R}$, even though the convergence is not uniform on $\mathbb{R}$. (You can assume this but try $\epsilon = 1$ to see why.)

5. Show that $f'_n \to h_0$ and that the convergence is uniform on $[0, 2]$. Hence verify that $\lim_{n\to\infty} f'_n = (\lim_{n\to\infty} f_n)'$.

6. Now try $g_n(x) = x + (\sin x)/n$ and go through the same steps.

7. Let $h_n(x) = nx$ for $0 \leq x \leq 1/n$ and $h_n(x) = n(1 - x)/(n - 1)$ for $n \in \mathbb{N}$ and $x \in [0, 1]$. Verify pointwise convergence to h_0 and then show, by explicit integration of h_n that this convergence is not uniform. Draw some pictures then to show what is going on.

29th September 2008