

MATH252A – Elements of Analysis and Algebra

TEST 1

Thursday 29 March 2007 - (55 mins) – Answer **ALL** questions

1. (a) Describe $\left\{x \in \mathbb{R} : \frac{x+1}{x-1} > 2\right\}$ as an interval.
(b) If $|x-3| < 1$ show $|x+2| < 6$ [Hint: $|x-3+5|$].
2. Give definitions for each of the following:
- (a) $\lim_{n \rightarrow \infty} a_n = L, L \in \mathbb{R},$ (b) $\lim_{n \rightarrow \infty} a_n = \infty,$
(c) $\sum_{n=1}^{\infty} a_n = s, s \in \mathbb{R},$ (d) $S \subset \mathbb{R}$ and $\alpha = \text{lub}(S).$
3. Estimate the value of the following limits and sums as real numbers or $\pm \infty$. There is no need to give proofs.
- (a) $\lim_{n \rightarrow \infty} \frac{3n^3 + n^2 + 1}{2n^3 + n + 1},$ (b) $\lim_{n \rightarrow \infty} \frac{\sin(n)}{n^2 + 1},$
(c) $\sum_{n=1}^{\infty} \left(\frac{1}{3^n} + \frac{1}{n}\right),$ (d) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}.$
4. If $a_n = \frac{n-1}{n+1}$ is a sequence, use ε and N_ε to prove that $\lim_{n \rightarrow \infty} a_n = 1$ by, given $\varepsilon > 0$, finding an explicit N_ε . Then prove this result by using limit theorems.
5. (a) Use Abel's Test or the Integral Test to show $\sum_{n=1}^{\infty} \frac{3}{4n+1}$ diverges.
(b) Use a comparison and D'Alembert's (Ratio) Test to show $\sum_{n=1}^{\infty} \frac{1}{2^n + n}$ converges.
(c) Use the Alternating Series Test to show $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converges, and then show it is **not** absolutely convergent.