UNIVERSITY OF WAIKATO
Department of Mathematics

MATH252-09B – Elements of Analysis

TEST 1

Wednesday 19 August 2009 - (55 mins) – Answer **ALL** questions

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1. Give definitions and statements for

   (a) $\lim_{n\to\infty} a_n = L$,

   (b) Abel's test for series divergence,

   (c) the integral test for series convergence,

   (d) $\lim_{x\to a+} f(x) = +\infty$.

2. Give the statement of the least upper bound axiom for the real numbers. Assume that if (a_n) is an decreasing sequence which is bounded below, then $\lim_{n\to\infty} a_n$ exists as a finite real number to show that if $0 < c < 1$ and $a_n = c^n$ for $n = 1, 2, 3$, then $\lim_{n\to\infty} a_n = 0$.

3. Let $\lim_{x\to a+} f(x) = L$ and $\lim_{x\to a+} g(x) = M$. Prove, using ε and δ_ε, that $\lim_{x\to a+} (f(x)+g(x)) = L + M$.

4. (a) If $a_n = \dfrac{4n+1}{2n+1}$ is a sequence, use ε and N_ε to prove that $\lim_{n\to\infty} a_n = 2$ by, given $\varepsilon > 0$, finding an explicit N_ε. Then prove this result by using limit theorems.

   (b) Using D'Alembert's limit test or otherwise, show that $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$ converges.

5. Let $f(x) = 2x + \dfrac{1}{x}$. Given $\varepsilon > 0$, find $\delta_\varepsilon > 0$ so that $|x-1| < \delta_\varepsilon \Rightarrow |f(x) - f(1)| < \varepsilon$.

   Hence show f is continuous at $x = 1$. Discuss the behaviour of $f(x)$ near $x = 0$ and as $x \to +\infty$.

   Sketch the graph of f.