UNIVERSITY OF WAIKATO
Department of Mathematics

MATH252-08B – Elements of Analysis

TEST 1

Wednesday 20 August 2008 - (55 mins) – Answer **ALL** questions

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1. Give definitions and statements for

3 (a) $\lim\limits_{x \to a+} f(x) = L$,

3 (b) $\lim\limits_{n \to \infty} a_n = \infty$,

3 (c) a form of D'Alembert's test for series convergence or divergence.

3 (d) the integral test for series convergence or divergence.

2. Give the statement of the least upper bound axiom for the real numbers. 4 Prove that if (a_n) is 9 an increasing sequence which is bounded above, then $\lim\limits_{n \to \infty} a_n$ exists as a finite real number. 9 Then, using this result, show that $\lim\limits_{n \to \infty} \left(2 - \frac{3}{n}\right) = 2$.

3. 12 Let $\lim\limits_{n \to \infty} a_n = L$ and $\lim\limits_{n \to \infty} b_n = M$. Prove, using ε and N_ε, that $\lim\limits_{n \to \infty} (a_n + b_n) = L + M$.

4. (a) 6 If $a_n = \dfrac{5n-2}{n+1}$ is a sequence, use ε and N_ε to prove that $\lim\limits_{n \to \infty} a_n = 5$ by, given $\varepsilon > 0$, finding an explicit N_ε. Then prove this result by using limit theorems. 2

   (b) 9 Using Abel's test or otherwise, show $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+6}$ diverges.

5. 6 Let $f(x) = 8x + \dfrac{2}{x} + 1$. Given $\varepsilon > 0$, find $\underset{\varepsilon}{\delta} > 0$ so that $|x-1| < \underset{\varepsilon}{\delta} \Rightarrow |f(x) - f(1)| < \varepsilon$.

   2 Hence show f is continuous at $x = 1$. 2 Discuss the behaviour of $f(x)$ near $x = 0$ and as $x \to +\infty$.

   Sketch the graph of f.