

The University of Waikato
Department of Mathematics

Elements of Analysis math252-10B Assignment 3 (Revised)

Assignment 3 is to do questions 4-7. It is due Thursday 23rd September.

1. Let $f(x) = x(x - 1)(x - 3)$. Sketch the graph of $y = f(x)$ on $[-1, 4]$.

2. Apply Rolle's theorem to $f(x)$ on $[0, 1]$ and then find a point $x = \xi$ such that $0 < \xi < 1$ and $f'(\xi) = 0$.

3. Apply the Mean Value theorem to $f(x)$ on $[1, 4]$ and then find all of the points $x = \xi$ in $(1, 4)$ such that $f'(\xi) = (f(4) - f(1))/(4 - 1)$.

4. Give the Taylor expansion for $f(x)$ about $x = 0$ in the form

$$f(0 + h) = f(0) + f'(0)h + \frac{1}{2!}f''(0 + \theta h)h^2$$

and then, if $h = 0.1$, find θ explicitly and verify $|\theta| < 1$.

5. For the four term plus remainder Taylor expansion for $f(x)$ about $x = a$, explain why the remainder term $f^4(a + \theta h)h^4/4!$ is always zero so

$$f(a + h) = f(a) + f'(a)h + \frac{1}{2!}f''(a)h^2 + \frac{1}{3!}f'''(a)h^3$$

exactly for all h, a . Then do this explicitly.

6. What is a point of inflection ? Find all the points of inflection of the given $f(x)$ and the slope of the tangent to the graph of $f(x)$ at the point or points, verifying the property.

7. Let $g(x) = x^4(x - 1)(x - 3)$. Find the nature of the critical point at $x = 0$. By expanding the definition, find the Taylor series for $g(x)$ about $x = 0$ including all terms, even when they are 0.

15th September 2010