math 252 - 10 B Assign 2 Worked Answers

(1) $\lim_{n\to\infty} \frac{n^2}{n^2+2} = \lim_{n\to\infty} \frac{1}{1+\frac{2}{n^2}} = \frac{\lim_{n\to\infty} 1}{\lim_{n\to\infty}(1+\frac{2}{n^2})} = \frac{1}{1+0} = 1$

using lt of a quot is the quot of the limits and lt of a sum is the sum of the lts.

Given $\varepsilon > 0$ working back we want $\left|\frac{n^2}{n^2+2} - 1\right| < \varepsilon \iff \left|\frac{n^2-n^2-2}{n^2+2}\right| < \varepsilon$

$\iff \left|\frac{-2}{n^2+2}\right| < \varepsilon \iff \frac{2}{n^2+2} < \varepsilon \iff \frac{n^2+2}{2} > \frac{1}{\varepsilon} \iff n^2 > \frac{2}{\varepsilon} - 2$

so let N_ε be a pos. integer with $N_\varepsilon > \sqrt{\frac{2}{\varepsilon}-2}$. Then $n \geq N_\varepsilon \Rightarrow$

$n > \sqrt{\frac{2}{\varepsilon}-2} \Rightarrow n^2 > \frac{2}{\varepsilon} - 2 \Rightarrow |a_n - 1| < \varepsilon$ //

(2) $\lim_{n\to\infty}\left((2n)^{\frac{1}{n}} + 5\frac{\log n}{n}\right) = \lim_{n\to\infty} 2^{\frac{1}{n}} n^{\frac{1}{n}} + 5\lim_{n\to\infty}\frac{\log n}{n}$

$= \lim_{n\to\infty} 2^{1/n} \lim_{n\to\infty} n^{1/n} + 5\lim_{n\to\infty}\frac{\log n}{n}$

$= 1.1 + 5.0 = 1.$ //

(3) $\sum_{n=1}^{3}\left(\frac{1}{4^n} + \frac{3}{n(n+1)}\right) = \left(\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \frac{3}{1.2} + \frac{3}{2.3} + \frac{3}{3.4}\right) = \frac{165}{64}$

$\sum_{n=1}^{n}\left(\frac{1}{4^n} + \frac{3}{n(n+1)}\right) = \left(\sum_{n=0}^{n}\frac{1}{4^n}\right) - 1 + 3\sum_{n=1}^{n}\left(\frac{1}{n} - \frac{1}{n+1}\right)$

$= \frac{1-\frac{1}{4^{n+1}}}{1-\frac{1}{4}} - 1 + 3\left(1 - \frac{1}{n+1}\right) = S_n.$

$\sum_{n=1}^{\infty}\left(\frac{1}{4^n} + \frac{3}{(n+1)n}\right) = \lim_{n\to\infty} S_n = \frac{1}{1-\frac{1}{4}} - 1 + 3 = \frac{4}{3} + 2 = \frac{10}{3}.$

(4) (a) $a_n = \frac{5^n}{5^n+1} = \frac{1}{1+\frac{1}{5^n}} \to \frac{1}{1+0} \neq 0 \therefore \sum_{n=1} a_n$ D.

(b) $a_n = \frac{1}{n^2+2n+3}$, $b_n = \frac{1}{n^2}$ then $0 < a_n < b_n$ and $\sum_{n=1} b_n$ C (since $n^{3/2} b_n \to 0$)

$\therefore \sum_{n=1}^{\infty} a_n$ C by the comparison test

(c) $a_n = \frac{4^n}{n!} \Rightarrow \frac{a_{n+1}}{a_n} = \frac{4^{n+1}}{(n+1)n!}\cdot\frac{n!}{4^n} = \frac{4}{n+1} \leq \frac{1}{2} \iff$

$8 \leq n+1$

$\iff 7 \leq n$

$\therefore$ By D'Alembert test $\sum_{n=7}^{\infty} a_n$ C $\therefore \sum_{n=1}^{\infty}$ C

(d) $a_n = \frac{1}{3n+2}$ is decreasing $\iff a_{n+1} < a_n \iff \frac{1}{3(n+1)+2} < \frac{1}{3n+2}$

$\iff 3n+3+2 > 3n+2 \iff 5 > 2$ ✓. But $na_n = \frac{n}{3n+2} \to \frac{1}{3} \neq 0.$

$\therefore$ By Abel's test $\sum_{n=1}^{\infty} a_n = \infty$ D. or $\int_1^T \frac{dx}{3x+2} = \frac{1}{3}\ln(3T+2) - \frac{1}{3}\ln 3$

$\to \infty$ //