

The University of Waikato  
Department of Mathematics

Elements of analysis and algebra math252-07A 2007 Assignment 2

Due Thursday 22nd March: Please hand back your completed assignment through the slot for this paper outside the Mathematics Office G3.19. (Neatly and on no more than four sides of an A4 page).

1. Sum the following series to four terms and to infinity. (There is no need to justify convergence.)

$$\sum_{n=2}^{\infty} \frac{3}{n(n-1)} + \frac{2}{4^n}$$

2. We say the sequences of positive terms  $(a_n)$  and  $(b_n)$  are “ordered” or  $(a_n) < (b_n)$  if  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0$ . Order the following sequences:

$$(n), (2^n), (e^n), (n!), (\log n), (n^n), (n^\alpha) (0 < \alpha < 1).$$

3. Use tests to determine whether each of the following series converges or diverges:

$$(a) \sum_{n=2}^{\infty} \frac{2}{n+1}, (b) \sum_{n=1}^{\infty} \frac{(\log n)^n}{n^n}, (c) \sum_{n=1}^{\infty} n!e^{-n}, (d) \sum_{n=1}^{\infty} \frac{3^n (n!)^2}{(2n)!}.$$

4. Prove that a series of positive terms converges to a finite real number if and only if its corresponding sequence of partial sums is bounded above.

15th March 2006