MATH252-10B Assign 1 Worked Answers

1) First $x \neq 1$ since division by 0 is not permitted. If $1-x>0$ then $\frac{1}{1-x} > \frac{1}{2} \Rightarrow$ $1-x<2 \Rightarrow -1<x$. Also $1-x>0 \Leftrightarrow x<1$ so we get $-1<x<1 \equiv (-1,1)$.
If $1-x<0$ then $\frac{1}{1-x} < 0 < \frac{1}{2}$ so there are no solution points.

2) If $x \geq 0$ $x = |x| \leq \varepsilon \Rightarrow x \leq \varepsilon \Rightarrow -\varepsilon \leq 0 \leq x \leq \varepsilon \Rightarrow -\varepsilon \leq x \leq \varepsilon$.
If $x \leq 0$ $-x = |x| \leq \varepsilon \Rightarrow -x \leq \varepsilon \Rightarrow -\varepsilon \leq x < 0 \leq \varepsilon \Rightarrow -\varepsilon \leq x \leq \varepsilon$

$\therefore$ in all cases $-\varepsilon \leq x \leq \varepsilon$.

3) $\lim_{n\to\infty} \left(\frac{n^2+1}{n^2+2}\right)\left(2-\frac{1}{n}\right) = \lim_{n\to\infty}\left(\frac{n^2+1}{n^2+2}\right)\lim_{n\to\infty}\left(2-\frac{1}{n}\right)$ Lt prod = prod. Lts.

$= \lim_{n\to\infty}\left(\frac{1+\frac{1}{n^2}}{1+\frac{2}{n^2}}\right)\left(\lim_{n\to\infty} 2 - \lim_{n\to\infty}\frac{1}{n}\right)$ Lt sum = sum lts.

$= \frac{\lim_{n\to\infty} 1+\frac{1}{n^2}}{\lim_{n\to\infty}\left(1+\frac{2}{n^2}\right)}(2-0) = \frac{1+0}{1+0}(2-0) = 2$ Lt quot = quot Lts.

4) $\forall n \in \mathbb{N}$ $-n \leq n\sin(n) \leq n \Leftarrow -1 \leq \sin(n) \leq 1$

$\therefore -\frac{n}{n^2+1} \leq \frac{n\sin(n)}{n^2+1} \leq \frac{n}{n^2+1}$ But $\frac{n}{n^2+1} \to 0$ so $\frac{-n}{n^2+1} \to 0$ also &

By the sandwich thm $\lim_{n\to\infty} \frac{n\sin(n)}{n^2+1} = 0$.

5) Given $\varepsilon > 0$. Working back we want $|a_n - 3| < \varepsilon \Leftrightarrow \left|\frac{6n-2}{2n+1} - 3\right| < \varepsilon$
$\Leftrightarrow \left|\frac{6n-2-6n-3}{2n+1}\right| < \varepsilon \Leftrightarrow \left|\frac{-5}{2n+3}\right| < \varepsilon \Leftrightarrow \frac{5}{2n+3} < \varepsilon \Leftrightarrow 2n+3 > \frac{\varepsilon}{5}$
$\Leftrightarrow n > \frac{\varepsilon}{10} - \frac{3}{2}$

So let N_ε be a pos. integer with $N_\varepsilon > \frac{\varepsilon}{10} - \frac{3}{2}$
Then $n \geq N_\varepsilon \Rightarrow |a_n - 3| < \varepsilon$ so $\lim_{n\to\infty} a_n = 3$.

6) $S = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\right\}$ and $S \subset [\frac{1}{2}, 1)$
Clearly $\frac{1}{2} \leq \frac{n}{n+1} \Leftrightarrow n+1 \leq 2n \Leftrightarrow 1 \leq n$ which is true for $n \in \mathbb{N}$. Hence $\frac{1}{2}$ is a l.b. If β was an number with $\frac{1}{2} < \beta$ it would not be a l.b.
Hence $\frac{1}{2}$ is the glb.
Clearly $\frac{n}{n+1} < 1$ ($\Leftarrow$ $n < n+1 \Leftrightarrow 0 < 1$) so 1 is an u.b. Given $\varepsilon > 0$ choose $n \in \mathbb{N}$ so $\frac{1}{n+1} < \varepsilon$. Then $-\varepsilon < -\frac{1}{n+1} \Rightarrow 1-\varepsilon < 1-\frac{1}{n+1} = \frac{n}{n+1} < 1$
& $\frac{n}{n+1} \in S$. Hence $1 = \text{lub}(S)$