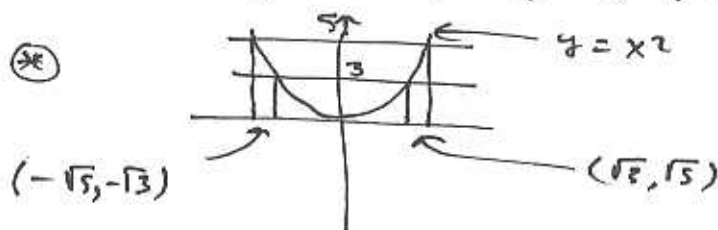


① $|x^2 - 4| < 1 \Leftrightarrow -1 < x^2 - 4 < 1 \Leftrightarrow 3 < x^2 < 5$ *

$x > 0 \Leftrightarrow \sqrt{3} < x < \sqrt{5}$ or
 $x < 0 \Leftrightarrow \sqrt{3} < -x < \sqrt{5} \Leftrightarrow -\sqrt{5} < x < -\sqrt{3}$

So $\{x : |x^2 - 4| < 1\} = (-\sqrt{5}, -\sqrt{3}) \cup (\sqrt{3}, \sqrt{5})$

OR by *



② $\lim_{n \rightarrow \infty} \left(\frac{n+1}{3n+2} \right) \left(6 + \frac{1}{n^2} \right) = \lim_{n \rightarrow \infty} \frac{n+1}{3n+2} \cdot \lim_{n \rightarrow \infty} \left(6 + \frac{1}{n^2} \right)$ (a)

$= \lim_{n \rightarrow \infty} \frac{1 + \frac{1}{n}}{3 + \frac{2}{n}} \cdot \left(\lim_{n \rightarrow \infty} 6 + \left(\lim_{n \rightarrow \infty} \frac{1}{n} \right)^2 \right)$ (b)

$= \frac{\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)}{\lim_{n \rightarrow \infty} \left(3 + \frac{2}{n} \right)} \left(6 + 0^2 \right)$ (c) $= \left(\frac{1+0}{3+0} \right) (6+0) = \frac{6}{3} = 2 //$

(a) L't. of a prod. is the prod. of the limits

(b) \div numerator and denominator by n , L't. of a sum is the sum of the L'ts.

(c) L't. of a quotient is the quotient of the L'ts.

$\forall n: -1 \leq \sin(n) \leq 1 \Rightarrow -1 \leq (-1)^n \sin(n) \leq 1$ also

Hence $-\frac{1}{n^2} \leq \frac{(-1)^n \sin(n)}{n^2} \leq \frac{1}{n^2}$ since $n^2 > 0$

$\Rightarrow 2 - \frac{1}{n^2} \leq \frac{(-1)^n \sin(n)}{n^2} + 2 \leq 2 + \frac{1}{n^2} \quad \forall n \in \mathbb{N}$. But $2 - \frac{1}{n^2} \rightarrow 2$ and $2 + \frac{1}{n^2} \rightarrow 2$

\Rightarrow , by the Squeeze Thm, $\frac{(-1)^n \sin(n)}{n^2} + 2 \rightarrow 2$ also.

Given $\epsilon > 0$. Want $|a_n - \frac{1}{2}| < \epsilon$. This is true

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$$\Leftrightarrow \left| \frac{n+1}{2n+2} - \frac{1}{2} \right| < \epsilon$$

$$\Leftrightarrow \left| \frac{2n+2-2n-1}{2(2n+2)} \right| < \epsilon$$

$$\Leftrightarrow \frac{1}{2(2n+2)} < \epsilon$$

$$\Leftrightarrow 2(2n+2) > \frac{1}{\epsilon}$$

$$\Leftrightarrow 2n+1 > \frac{1}{2\epsilon} \Leftrightarrow n > \frac{1}{4\epsilon} - \frac{1}{2}$$

So let N_ϵ be any element of \mathbb{N} with $N_\epsilon > \frac{1}{4\epsilon} - \frac{1}{2}$. So if

$n > N_\epsilon$ we have $n > N_\epsilon > \frac{1}{4\epsilon} - \frac{1}{2} \Rightarrow n > \frac{1}{4\epsilon} - \frac{1}{2}$ which

is equivalent to $|a_n - \frac{1}{2}| < \epsilon$. Hence $a_n \rightarrow \frac{1}{2}$ //

$$a_n = \frac{2^n - 1}{2^n} = \frac{2^n}{2^n} - \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

$$a_{n+1} > a_n \Leftrightarrow 1 - \frac{1}{2^{n+1}} > 1 - \frac{1}{2^n}$$

$$\Leftrightarrow \frac{1}{2^n} > \frac{1}{2^{n+1}} = \frac{1}{2^n} \cdot \frac{1}{2}$$

$$\Leftrightarrow 1 > \frac{1}{2} \text{ which is true. Hence } (a_n) \text{ is increasing.}$$

(So (a_n) is bounded above by 1.

$$a_n = 1 - \frac{1}{2^n} < 1 \Leftrightarrow -\frac{1}{2^n} < 0 \Leftrightarrow 0 < \frac{1}{2^n} \text{ which is true.}$$

Hence $\lim_{n \rightarrow \infty} a_n \exists$.

To calculate the limit:

$$\begin{aligned} \lim_{n \rightarrow \infty} a_n &= \lim_{n \rightarrow \infty} \left(1 - \frac{1}{2^n} \right) \\ &= 1 - \lim_{n \rightarrow \infty} \left(\frac{1}{2} \right)^n \\ &= 1 - 0 = 1 \end{aligned}$$

From the useful limits

$$c^n \rightarrow 0 \text{ when } |c| < 1 //$$