The University of Waikato
Department of Mathematics

Elements of Analysis and Algebra math252-07A 2007 Assignment 1

Due Thursday 15th March: Please hand back your completed assignment through the slot for this paper outside the Mathematics Office G3.19.

It should be written up neatly and on no more than four sides of an A4 page or the equivalent.

1. Express the set $\{x : |x^2-4| < 1\}$ as the union of two open intervals OR (harder) prove that $\sqrt{3}$ is not a rational number and generalize to $\sqrt{n}$ for $n \in \mathbb{N}$ being as large a set as possible.

2. Use limit theorems to prove that

$$\lim_{n\to\infty} (\frac{n+1}{3n+2})(6+\frac{1}{n^2})=2.$$

3. Use the sandwich theorem to prove that

$$\lim_{n\to\infty}(\frac{(-1)^n \sin n}{n^2}+2) = 2.$$

4. Let a sequence (a_n) be defined by

$$a_n = \frac{n+1}{2n+1}$$

Given $\varepsilon > 0$ find an $N_\varepsilon \in \mathbb{N}$ such that

$$\left|a_n - \frac{1}{2}\right| < \varepsilon$$

for all n with $N_\varepsilon \leq n$.
Hence prove that $a_n \to \frac{1}{2}$.

5. Let the sequence (a_n) be defined by $a_n = (2^n-1)/2^n$. Prove that the sequence is increasing and bounded above. Deduce that the limit exists. Find the limit using the limit theorem.

Kevin Broughan 8th March 2007