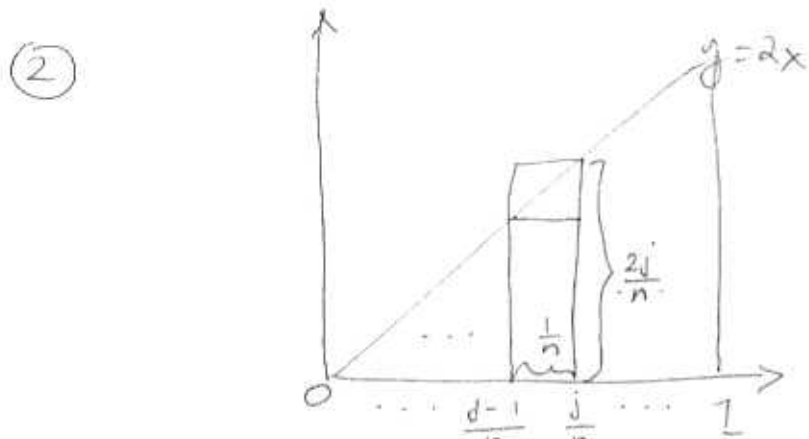


math 252-09 B Assign 4

(1) $T = (2, 3)$ then $\sup(-T) = \sup((-3, -2)) = -2$
 $\inf(T) = \inf((2, 3)) = 2 \Rightarrow$
 $\Rightarrow -2 = \sup(-T) = -\inf(T)$ //



$$L(f, P_n) = \sum_{j=1}^n \frac{2(j-1)}{n} \cdot \frac{1}{n} = \frac{2}{n^2} \sum_{j=1}^n j - \sum_{j=1}^n \frac{2}{n^2} = \frac{2}{n^2} \frac{n(n+1)}{2} - \frac{2}{n^2} \cdot n$$

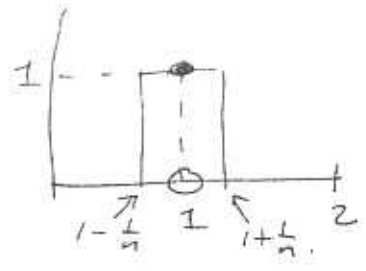
$$= \frac{n+1}{n} - \frac{2}{n} = 1 + \frac{1}{n} - \frac{2}{n} = 1 - \frac{1}{n}$$

$$U(f, P_n) = \sum_{j=1}^n \frac{2j}{n} \cdot \frac{1}{n} = \frac{2}{n^2} \sum_{j=1}^n j = \frac{2}{n^2} \frac{n(n+1)}{2} = \frac{n+1}{n} = 1 + \frac{1}{n}$$

$$U(f, P_n) - L(f, P_n) = (1 + \frac{1}{n}) - (1 - \frac{1}{n}) = \frac{2}{n} = \epsilon_n \rightarrow 0 \therefore f \text{ is Riemann-Int.}$$

hence \downarrow
 $L(f, P_n) \leq \int_0^1 f \leq U(f, P_n) \Rightarrow 1 - \frac{1}{n} \leq \int_0^1 f \leq 1 + \frac{1}{n} \quad \forall n \rightarrow \infty$

$$\Rightarrow 1 + 0 \leq \int_0^1 f \leq 1 + 0 \Rightarrow \int_0^1 f = 1 //$$



(3) $P_n = (0, 1 - \frac{1}{n}, 1 + \frac{1}{n}, 2)$

$$L(f, P_n) = 0 \cdot \Delta x_1 + 0 \cdot \Delta x_2 + 0 \cdot \Delta x_3 = 0$$

$$U(f, P_n) = 0 \cdot \Delta x_1 + 1 \cdot (1 + \frac{1}{n} - (1 - \frac{1}{n})) + 0 \cdot \Delta x_3$$

$$= \frac{2}{n}$$

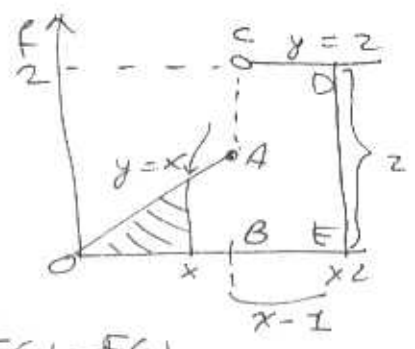
Hence $U(f, P_n) - L(f, P_n) = \frac{2}{n} \rightarrow 0 \Rightarrow$

$$0 \leq \int_0^2 f \leq \frac{2}{n} \Rightarrow, \text{ as } n \rightarrow \infty \text{ so } \int_0^2 f = 0 //$$

f is Riemann-Int. \downarrow

(4) If $0 \leq x \leq 1$, $F(x) = \int_0^x f = \frac{1}{2} x x = \frac{x^2}{2}$
 If $1 < x$, $F(x) = \text{area } \Delta OAB + \text{rect } BCDE$

$$\Rightarrow F(x) = \begin{cases} \frac{x^2}{2}, & x \leq 1 \\ 2x - \frac{3}{2}, & x > 1 \end{cases} \Rightarrow F'(x) = \begin{cases} x & x < 1 \\ 2 & x > 1 \end{cases}$$



$$\lim_{x \rightarrow 1^-} F(x) = \lim_{x \rightarrow 1^-} \frac{x^2}{2} = \frac{1}{2} = \lim_{x \rightarrow 1^+} (2x - \frac{3}{2}) = \lim_{x \rightarrow 1^+} F(x) = F(1) //$$