The University of Waikato Department of Mathematics
Elements of Analysis Assignment 4: math252-09B

Assigment 4 is due Monday 5th October. Attempt questions 1,2,3 and 4.

Riemann Integration:

1. Let $T = (2, 3)$. Prove that $\sup(-T) = -\inf T$.

2. By using the uniform partition $(P_n : n \in \mathbb{N})$ and explict lower and upper sums show that the function $f(x) = 2x$ is Riemann integrable on $[0, 1]$ by verifying $U(f, P_n) - L(f, P_n) \leq \epsilon_n$ where ϵ_n is some explicit function of n which tends to 0 as $n \to \infty$. Then find the value of the integral using

$$L(f, P_n) \leq \int_0^1 f \leq U(f, P_n).$$

3. Let $f(x) = 0$ for $0 \leq x < 1$ and $1 < x \leq 2$ and $f(1) = 1$. By using the partition $P_n = (0, 1 - 1/n, 1 + 1/n, 2)$ show that f is Riemann integrable and evaluate $\int_0^2 f$.

4. Define $f : [0, 2] \to \mathbb{R}$ by $f(x) = x$ if $x \leq 1$ and $f(x) = 2$ if $x > 1$. Evaluate (using areas say) $F(x) = \int_0^x f$ as an explicit function of x and show explicitly it is continuous at $x = 1$. Verify that $F'(x) = f(x)$ when $x \neq 1$.

28th September 2009