

$$f(x) = x(x-1)(x-2) = x(x^2 - 3x + 2) = x^3 - 3x^2 + 2x$$

$$f(x) = x^3 - 3x^2 + 2x$$

$$f'(x) = 3x^2 - 6x + 2$$

$$f''(x) = 6x - 6$$

$$f(a+h) = f(a) + f'(a)h + \frac{1}{2!} f''(a+\theta h) h^2$$

$$h^3 - 3h^2 + 2h = 0 + 2h + \frac{1}{2!} (6\theta h - 6) h^2$$

$$\Rightarrow h^3 - 3h^2 = \frac{6}{2} (\theta h - 1) h^2$$

$$h \neq 0 \quad h - 3 = 3(\theta h - 1) \quad \Rightarrow \quad \theta = \frac{1}{3} \quad \text{i.e.} \quad |\theta| = \frac{1}{3} < 1$$

$\forall h \neq 0$

\therefore for $h = 0.1$

$$5) \quad f(a+h) = f(a) + f'(a)h + \frac{f''(a)}{2!} h^2 + \frac{f'''(a)}{3!} h^3 + \frac{f^{(iv)}(a+\theta h)}{4!} h^4$$

Since $f'''(x) = 6$ & $f^{(iv)}(x) = f^{(iv)}(x) = 0 \Rightarrow f^{(iv)}(a+\theta h) = 0$

$$\Rightarrow f(a+h) = f(a) + f'(a)h + \frac{f''(a)}{2!} h^2 + \frac{f'''(a)}{3!} h^3$$

Explicitly

$$\begin{aligned} \text{LHS} &= (a+h)^3 - 3(a+h)^2 + 2(a+h) \\ &= a^3 + 3a^2h + 3ah^2 + h^3 - 3(a^2 + 2ha + h^2) - 2(a+h) \\ &= a^3 + 3a^2h + 3ah^2 + h^3 - 3a^2 - 6ah - 3h^2 - 2a - 2h \end{aligned}$$

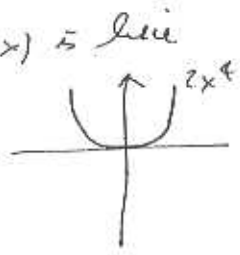
$$\begin{aligned} \text{RHS} &= f(a) = a^3 - 3a^2 + 2a \\ &+ f'(a)h = + h(3a^2 - 6a + 2) = + 3a^2h - 6ah + 2h \\ &+ \frac{f''(a)}{2!} h^2 = + \frac{6h^2}{2} (a-1) = + 3ah^2 - 3h^2 \\ &+ \frac{f'''(a)}{3!} h^3 = + \frac{6h^3}{6} (1) = + h^3 \end{aligned}$$

and $\text{LHS} = \text{RHS}$

A point of inflection is where the tangent to a curve crosses the curve i.e. on one side it is below and on the other above the curve.

Here $f(x) = x^3 - 3x^2 + 2x \Rightarrow f''(x) = 6x - 6, f'''(x) = 6$
Then $f''(1) = 0$ and $f'''(1) = 6 > 0$ so the first non vanishing derivative, other than the first, is of odd order $\Rightarrow x=1, y=f(1)$ is a point of inflection. Since $f^{(k)}(x) = 0 \forall x \in \mathbb{R}$ there are no higher order pts of inflection. Note that $f'(1) = 3(1)^2 - 6(1) + 2 \neq 0$, so the pt. of inflection is not horizontal.

7) Let $g(x) = x^4(x-1)(x-2)$

Near $x=0$ $g(x) \approx x^4(0-1)(0-2) = 2x^4$ so $g(x)$ is like 

$\therefore x=0$ gives a local minimum.

OR $g(x) = x^4(x^2 - 3x + 2) = x^6 - 3x^5 + 2x^4 \Rightarrow g(0) = 0$
 $g'(x) = 6x^5 - 15x^4 + 8x^3 \Rightarrow g'(0) = 0$
 $g''(x) = 30x^4 - 12x^3 + 24x^2 \Rightarrow g''(0) = 0$
 $g'''(x) = 4 \cdot 30x^3 - 3 \cdot 12x^2 + 2 \cdot 24x \Rightarrow g'''(0) = 0$
 $g^{(4)}(x) = 3 \cdot 4 \cdot 30x^2 - 2 \cdot 3 \cdot 12x + 2 \cdot 24 \Rightarrow g^{(4)}(0) = 48 > 0$
since the first non vanishing derivative is positive \Rightarrow local min.

Taylor Series

$$g(x) = x^6 - 3x^5 + 2x^4 = 0 + 0 \cdot x + 0 \cdot x^2 + 0 \cdot x^3 + 2x^4 + (-3)x^5 + x^6 + 0 + 0$$

is the Taylor series for $g(x)$.