Elements of Analysis   math 252-09B   Ass 2 Solutions

① $\lim_{n\to\infty} \frac{\log(n^{100})}{n} = \lim_{n\to\infty} 100\frac{\log(n)}{n} = 100\lim_{n\to\infty}\frac{\log n}{n} = 100\times 0 = 0.$ //

② $\lim_{n\to\infty}\left[(2n)^{\frac{1}{n}} + 5\left(1+\frac{1}{2n}\right)^{4n}\right] = \lim_{n\to\infty} 2^{\frac{1}{n}}.n^{\frac{1}{n}} + 5\lim_{n\to\infty}\left(1+\frac{1}{2n}\right)^{2n.2}$

$= \lim_{n\to\infty} 2^{\frac{1}{n}}.\lim_{n\to\infty} n^{\frac{1}{n}} + 5\Big[\lim_{\substack{m\to\infty \\ m=2n}}\left(1+\frac{1}{m}\right)^{m}\Big]^2$ since "lt. of sum = sum of lts" & "lt. of a prod. = prod of lts".

$= 1.1 + 5e^2$ using the "useful limits".

$= 1+5e^2$ //

③ $s_3 = 2\frac{3^2}{5^1} + 2.\frac{3^3}{5^2} + 2\frac{3^4}{5^3} = \frac{882}{125} = 7.056$ //

$\sum_{n=1}^{\infty} 2\frac{3^{n+1}}{5^n} = 6\sum_{n=1}^{\infty}\left(\frac{3}{5}\right)^n = 6\times\frac{3}{5}\sum_{n=0}^{\infty}\left(\frac{3}{5}\right)^n = \frac{18}{5}\frac{1}{1-\frac{3}{5}}$

$= \frac{18}{2} = 9$ //

④ $a_n = \frac{n^3-3n+2}{n(n+1)(n+2)} + \frac{1}{n^3}$

$= \frac{\frac{n^3}{n^3} - \frac{3n}{n^3} + \frac{2}{n^3}}{\frac{n}{n}\left(\frac{n+1}{n}\right)\left(\frac{n+2}{n}\right)} + \frac{1}{n^3} = \frac{1 - \frac{3}{n^2} + \frac{2}{n^3}}{1\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)} + \frac{1}{n^3}$

$\therefore \lim_{n\to\infty} a_n = \frac{\lim_{n\to\infty}\left(1-\frac{3}{n^2}+\frac{2}{n^3}\right)}{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\lim_{n\to\infty}\left(1+\frac{2}{n}\right)} + \lim_{n\to\infty}\frac{1}{n^3}$

since "lt. of a quot. is the quot. of the lts" etc)

$= \frac{1-0+0}{(1+0)(1+0)} + 0 = 1 \neq 0$

So $\lim_{n\to\infty} a_n \neq 0 \quad \therefore \quad \sum_{n=1}^{\infty} a_n = \infty$ i.e. $\mathcal{D}$. //