The University of Waikato Department of Mathematics
Elements of Analysis Workshop/Tutorial 5: math252-08B

For the Workshop/Tutorial on 10th September, attempt questions 1,2,3 and 4.

Riemann Integration:

1. By using the uniform partition $(P_n : n \in \mathbb{N})$ and explict lower and upper sums show that the function $f(x) = 2x$ is Riemann integrable on $[0, 1]$ by verifying $U(f, P_n) - L(f, P_n) \leq \epsilon_n$ where ϵ_n is some explicit function of n which tends to 0 as $n \to \infty$. Then find the value of the integral using

$$L(f, P_n) \leq \int_0^1 f \leq U(f, P_n).$$

2. Let $f(x) = 0$ for $0 \leq x < 1$ and $1 < x \leq 2$ and $f(1) = 1$. By using the partition $P = (0, 1, 1, 2)$ show that f is Riemann integrable and evaluate $\int_0^2 f$. Generalize this result to a function which is non-zero at at most a finite number of points $\{a_1, \cdots, a_n\}$ on an interval $[a, b]$.

3. Define $q(x) = 1$ if $x \in \mathbb{Q}$ and $g(x) = 0$ otherwise. Prove that q is not Riemann integrable on $[0, 1]$ by showing that for all partitions P,

$$U(q, P) - L(q, P) \geq 1$$

.

4. Construct a Riemann integrable function which is non-zero at an infinite number of points, is positive or zero everywhere, but which integrates to zero. Hint: $f(1/n) = 1, n \in \mathbb{N}$ and $f(x) = 0$ otherwise.

10th September 2008