

UNIVERSITY OF WAIKATO  
Department of Mathematics

***MATH101-10B - Introduction to Calculus***

**TEST 2**

**Thursday 7 October 2010**

**Time Allowed:** 1 hour and 20 mins

**Part A – TEN Short Answer** questions worth 5% each (50% of the total marks) - you should not spend more than about half the time on this.

**Part B – TWO Long Answer** questions worth 25% each (50% of the total marks).

**No one is to leave** the lecture room **during the last 10 minutes** of the test period.

Calculators may be used but **NO Graphical or Symbolic calculators permitted.**

---

**PART A**

***SHORT Answer Questions - worth 5% each***  
**(You do not need to show your working)**

1. If  $f(x) = x + \sin(x) + 1$  find  $\int f(x) dx$ .
2. Find the area under the graph  $y = 2 + 3x^2$  and above  $[1, 2]$ .
3. Given  $S(n) = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n}{6}(2n+1)(n+1)$  evaluate  $S(24)$  and  $\lim_{n \rightarrow \infty} \frac{S(n)}{n^3}$ .
4. If  $y = \sin\left(x + \frac{\pi}{4}\right)$  find  $\frac{dx}{dy}$  at  $y = \frac{1}{\sqrt{2}}$ .
5. Evaluate  $\int 3(x^2 + 1)^2 dx$ .
6. Evaluate  $\int_0^1 \frac{2x}{x^2 + 1} dx$ .
7. Find the antiderivative  $\int \left( \frac{1}{x+1} + \frac{1}{\sqrt{1-x^2}} + e^{2x} \right) dx$ .
8. Evaluate  $F'(x)$  if  $F(x) = \int_0^{x^2} \frac{e^t}{t} dt$ .
9. Find  $A$  and  $B$  if  $\frac{1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$ .
10. Evaluate  $\int_1^2 x \ln(x) dx$ .

**OVER ...**

## PART B

**LONG Answer Questions - worth 25% each**  
**(All working should be given)**

1. (a) Let  $f(x)$  be non-zero and differentiable. Prove the logarithmic differentiation formula i.e. that

$$(\ln|f(x)|)' = \frac{f'(x)}{f(x)}.$$

Hence evaluate the derivative of

$$\frac{(x+2)(x^3+2)^{1/2}}{(x+5)^2}$$

at  $x = 0$ .

- (b) Find the derivative of  $\ln|\sec(x) + \tan(x)|$  and hence show  
 $\int \sec(x) dx = \ln|\sec(x) + \tan(x)| + C$ .

- (c) Find  $A$  and  $B$  so  $\int e^x \sin(2x) dx = Ae^x \sin(2x) + Be^x \cos(2x) + C$ .

2. (a) Evaluate  $\int_0^1 \frac{(x+1)}{(x+2)(x+3)} dx$ .

- (b) Evaluate  $\int_0^1 \frac{x^2+2}{x^2+1} dx$ .

- (c) A solid has a circular base of radius 4 units. Find the volume of the solid which is such that every plane section perpendicular to a fixed diameter is an equilateral triangle.