

Part B (2008 Test 2) Math 101 Solutions

$$\textcircled{1} \text{ (a)} \quad \int_1^2 \left(x^3 + 2x^2 + \frac{1}{x} \right) dx = \left(\frac{x^4}{4} + \frac{2x^3}{3} + \ln|x| \right) \Big|_1^2$$

$$= \left(\frac{2^4}{4} + \frac{2 \cdot 2^3}{3} + \ln(2) \right) - \left(\frac{1^4}{4} + \frac{2 \cdot 1^3}{3} + \ln(1) \right)$$

$$= \frac{101}{12} + \ln(2) //$$

$$\text{(b)} \quad \text{Let } u = 1 - x^2 \Rightarrow du = -2x dx$$

$$\text{So } \int \frac{2x dx}{\sqrt{1-x^2}} = \int \frac{-1}{\sqrt{u}} du = - \int u^{-\frac{1}{2}} du$$

$$= -2u^{\frac{1}{2}} + C$$

$$= -2\sqrt{1-x^2} + C$$

$$\text{(c)} \quad I = \int \frac{x}{(x-1)(x-2)} dx \quad \text{Let } \frac{x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

$$\Rightarrow x = A(x-2) + B(x-1)$$

$$x=1 \Rightarrow 1 = A(-1) + 0 \Rightarrow A = -1$$

$$x=2 \Rightarrow 2 = 0 + B(1) \Rightarrow B = 2$$

$$\therefore I = \int \left(\frac{-1}{x-1} + \frac{2}{x-2} \right) dx$$

$$= - \int \frac{dx}{x-1} + 2 \int \frac{dx}{x-2} = -\ln|x-1| + 2\ln|x-2| + C$$

$$= \ln \left(\frac{|x-2|^2}{|x-1|} \right) + C //$$

(1) (d) $\cos(2x) = 2\cos^2 x - 1 \Rightarrow$

$\cos^2(x) = \frac{1}{2} + \frac{\cos(2x)}{2}$ \boxtimes

so $\cos^4(x) = \frac{1}{4} (1 + \cos(2x))^2$
 $= \frac{1}{4} (1 + 2\cos(2x) + \cos^2(2x))$

using \boxtimes with $x \rightarrow 2x$: $= \frac{1}{4} (1 + 2\cos(2x) + \frac{1}{2} + \frac{\cos(4x)}{2})$
 $= \frac{1}{8} (2 + 4\cos(2x) + 1 + \cos(4x))$

$\therefore \int_0^\pi \cos^4 x dx = \frac{1}{8} \int_0^\pi (3 + 4\cos(2x) + \cos(4x)) dx$
 $= \frac{1}{8} \left[3x + \frac{4\sin(2x)}{2} + \frac{\sin(4x)}{4} \right]_0^\pi$
 $= \frac{1}{8} [3\pi - 3 \cdot 0]$ since $\sin(0) = \sin(2\pi) = \sin(4\pi) = 0$
 $= \frac{3\pi}{8}$

(2) (a) $|f(x)| = |x| |x^2 + 1|^{2/3} |x + 4|^{-1} \Rightarrow (\log = \ln)$

$\log |f(x)| = \log|x| + \frac{2}{3} \log|x^2 + 1| - \log|x + 4|$

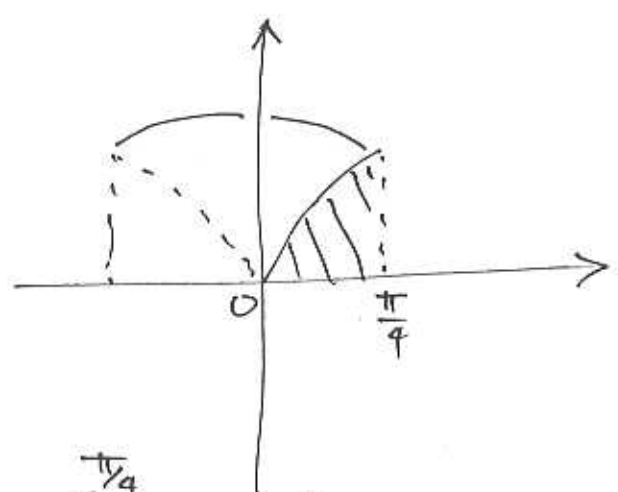
$\Rightarrow \frac{f'(x)}{f(x)} = \frac{(x)'}{x} + \frac{2}{3} \frac{(x^2 + 1)'}{x^2 + 1} - \frac{(x + 4)'}{x + 4}$
 $= \frac{1}{x} + \frac{4x}{3(x^2 + 1)} - \frac{1}{x + 4}$

$\Rightarrow f'(x) = f(x) \left[\frac{1}{x} + \frac{4x}{3(x^2 + 1)} - \frac{1}{x + 4} \right]$

This is well defined if $x \neq 0$ and $x \neq -4$.

$f'(1) = f(1) \left[\frac{1}{1} + \frac{4}{3} - \frac{1}{5} \right]$
 $= \frac{1 \cdot (2)^{2/3}}{5} \cdot \frac{22}{15} = \frac{22(2^{2/3})}{75}$

2 (b).



$f(x) = \sin(x)$

$$\text{Vol} = \int_0^{\pi/4} 2\pi x f(x) dx$$

$$= 2\pi \int_0^{\pi/4} \underbrace{x}_u \underbrace{\sin(x) dx}_{dv}$$

so let $u = x \Rightarrow du = dx$
 $\frac{dv}{dx} = \sin(x) \Rightarrow v = -\cos(x)$

$$\int_0^{\pi/4} x \sin(x) dx = uv \Big|_0^{\pi/4} - \int_0^{\pi/4} v du$$

$$= -x \cos(x) \Big|_0^{\pi/4} - \int_0^{\pi/4} (-\cos(x)) dx$$

$$= \left(-\frac{\pi}{4} \cos \frac{\pi}{4}\right) - (-0 \cos(0)) + \int_0^{\pi/4} \cos(x) dx$$

$$= -\frac{\pi}{4} \frac{\sqrt{2}}{2} + 0 + \sin(x) \Big|_0^{\pi/4}$$

$$= -\frac{\pi \sqrt{2}}{8} + \sin\left(\frac{\pi}{4}\right) - \sin(0)$$

$$= \frac{\sqrt{2}}{2} - \frac{\pi \sqrt{2}}{8}$$

$\therefore \text{Vol} = \frac{2\pi \sqrt{2}}{2} \left[1 - \frac{\pi}{4}\right] = \pi(4 - \pi)\sqrt{2} / 4$

(c) $\int_0^1 x^2 e^x dx = uv \Big|_0^1 - \int_0^1 v du$ where $u = x^2, dv = e^x dx \Rightarrow v = e^x$

$$= x^2 e^x \Big|_0^1 - \int_0^1 2x e^x dx$$

$$= e - 0 - 2 \int_0^1 x e^x dx$$

now let $u = x, dv = e^x dx$

$$= e - 2 \left[x e^x \Big|_0^1 - \int_0^1 e^x dx \right] = e - 2 \left[e - (e - 1) \right]$$

$$= e - 2$$