UNIVERSITY OF WAIKATO

Department of Mathematics

MATH101A - Introduction to Calculus

TEST 2

Wednesday 28 May 2008 - 7.05 pm **Time Allowed: 1 hour**

Part A - Answer questions on the *ANSWER SHEET* provided.
This is worth **48%** of the total marks and you should not spend more than about half the time on it.

Part B - Answer the **2 questions** in order. This is worth **52%** of the total marks.

No one is to leave the lecture room **during the last 15 minutes** of the test period.

Calculators (**NOT** programmable) may be used. Two pages of formulas are available.

PART A

○ = correct answer
□ = near answer

MULTI-CHOICE (each question is worth 4%)

1. Taylor's Theorem of the second order is

(A) $f(a+h)=f(a)+hf'(a)+\frac{h^2}{2}f''(a)$ (B) $f(a+h)=f(a)+hf'(a)+h^2f''(c)$

(C) $f(a+h)=f(a)+hf'(c)$ (D) $f(a+h)=f(a)+hf'(a)+\frac{h^2}{2}f''(c)$

(E) None of these.

2. If $f:[0,4]\to\mathbb{R}$ is defined by $f(x)=x^2+3x$ and $f'(c)=\frac{f(4)-f(0)}{4-0}$ then $c=$

(A) 1 (B) 2 (C) 3

(D) slope of the tangent (E) None of these.

3. Which of the following is an antiderivative for $\frac{1}{2}x^2-\frac{1}{x}$

(A) $x+\frac{1}{x^2}$ (B) $\frac{x^3}{2}-\frac{1}{x^2}$ (C) $\frac{x^3}{6}-\ln(|x|)$

(D) $\frac{x^3}{6}+\ln(|x|)$ (E) None of these.

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4. $\int_0^1 (2x+\sin(\pi x))dx =$

(A) 1 (A) 1 | (B) 3 | (C) $1+\frac{2}{\pi}$

(D) 4 | (E) None of these.

5. The expression $\sum_{j=1}^{n} 2\pi h(\overline{x}_j)(\overline{x}_j^2+1)\Delta x_j$ represents a Riemann sum for the integral of a function $f(x)$. The function is

(A) $2\pi h\left(\frac{x}{2}\right)x^2$ | (B) $2\pi(x^2+1)h(x)$ | (C) $\int_0^x f(t)dt$

(D) $2\pi h(x)(x^2+1)$ | (E) None of these.

6. If $F(x)=\int_0^{x^2} e^{t^2}dt$ then $F'(1)=$

(A) 0 | (B) e | (C) $2e$

(D) e^{x^2} | (E) None of these.

7. The surface area of the solid of revolution when the curve $y=f(x)$ is revolved about the $x-axis$ between $x=a$ and $x=b$ where f and f' are continuous on $[a,b]$, is given by

(A) $2\pi\int_a^b f(x)\sqrt{1+(f'(x))^2}dx$ | (B) $\pi\int_a^b (f(x))^2dx$ | (C) $2\pi\int_a^b xf(x)dx$

(D) $\int_a^b 2\pi f(x)\sqrt{1+f'(x)}dx$ | (E) None of these.

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8. Which of the following identities is **NOT** correct?

(A) $\cos(2\theta) = 2\cos^2(\theta) - 1$

(B) $\tan(2\theta) = \dfrac{2\tan\frac{\theta}{2}}{1 - \tan^2\left(\frac{\theta}{2}\right)}$

(C) $\tan^2(\theta) = \sec^2(\theta) + 1$

(D) $\sin^4(\theta) = (1 - \cos^2(\theta))^2$

(E) $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$.

9. If $f : \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right] \to \mathbb{R}$ is defined by $f(x) = \sin(x)$ then $\left(f^{-1}\right)'(x)$ is

(A) $\dfrac{1}{\sqrt{x^2 - 1}}$

(B) $\dfrac{-1}{\sqrt{1 - x^2}}$

(C) $\cos(x)$

(D) $\dfrac{1}{\sqrt{1 + x^2}}$

(E) None of these.

10. If $\int e^{-x}\cos(x)dx = Ae^{-x}\cos x + Be^{-x}\sin x + C$ then

$-\frac{1}{2}$ $\frac{1}{2}$

(A) $A = -1/2$ and $B = -1/2$

(B) $A = -1/2$ and $B = 1/2$

(C) $A = -1$ and $B = 0$

(D) $A = 1/4$ and $B = -1/4$

(E) None of these.

11. If $\dfrac{x^2 + 3x}{(x^2 - 1)(x + 2)}$ is split into partial fractions the form is

(A) $\dfrac{A}{x^2 - 1} + \dfrac{B}{x + 2}$

(B) $\dfrac{Ax + B}{x^2 - 1} + \dfrac{B}{x + 2}$

(C) $\dfrac{A}{x - 1} + \dfrac{B}{x + 1} + \dfrac{C}{x + 2}$

(D) $\dfrac{x^2}{(x^2 - 1)(x + 2)} + \dfrac{3x}{(x^2 - 1)(x + 2)}$

(E) None of these.

12. If $\int (4x - 2x)^{3/2} dx$ is simplified by completing the square the substitution would be

(A) $x = 2\tan\theta$

(B) $x = 1 + 2\sin\theta$

(C) $x - 1 = \sin(\theta)$

(D) $x - 1 = \sec(\theta)$

(E) None of these.

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PART B

(Each question is worth 26%)

1. Evaluate the following integrals

(a) $\displaystyle\int_1^2 \left(x^3 + 2x^2 + \frac{1}{x}\right) dx$

(b) $\displaystyle\int \frac{2x}{\sqrt{1-x^2}} dx$

(c) $\displaystyle\int \frac{x}{(x-1)(x-2)} dx$

(d) $\displaystyle\int_0^{\pi} \cos^4 x dx$.

2. (a) Use logarithmic differentiation to find an expression for the derivative of

$$f(x) = \frac{x(x^2+1)^{2/3}}{(x+4)}$$

Simplify your expression and note that values of x for which it is well defined. Compute $f'(1)$.

(b) The area under the graph of the function $f(x) = \sin(x)$ from $x = 0$ to $x = \pi/4$ is revolved about the $y-$axis. Use the method of shells and integration by parts to calculate the volume of the solid of revolution. There is no need to derive the integral formula you use.

(c) Evaluate $\displaystyle\int_0^1 x^2 e^x dx$.