

Part B

① (a) Let $u = x^2 + 1 \Rightarrow du = 2x dx \Rightarrow$

$$\begin{aligned} \int 2x(x^2+1)^4 dx &= \int (x^2+1)^4 2x dx \\ &= \int u^4 du = \frac{u^5}{5} + C \\ &= \frac{(x^2+1)^5}{5} + C \end{aligned}$$

(b) $\int \frac{x}{\sqrt{1-x^2}} dx = \int \frac{du}{\sqrt{u}} \quad \left\{ \begin{array}{l} u = 1-x^2 \\ -\frac{1}{2} du = x dx \end{array} \right.$

$$\begin{aligned} &= -\frac{1}{2} \int u^{-1/2} du \\ &= -\frac{1}{2} 2u^{1/2} + C \\ &= -u^{1/2} + C = -\sqrt{1-x^2} + C \end{aligned}$$

(c) Let $u = y + 1 \Rightarrow du = dy \leftarrow y = u - 1$

$$\begin{aligned} \int \frac{y}{\sqrt{y+1}} dy &= \int \frac{u-1}{\sqrt{u}} du = \int \left(\frac{u}{u^{1/2}} - \frac{1}{u^{1/2}} \right) du \\ &= \int (u^{1/2} - u^{-1/2}) du = \frac{2u^{3/2}}{3} - 2u^{1/2} + C \\ &= \frac{2}{3}(y+1)^{3/2} - 2(y+1)^{1/2} + C \end{aligned}$$

(d) $\int \sin^3(x) dx = \int \sin^2(x) \sin(x) dx \quad \left\{ \begin{array}{l} u = \cos x \\ -du = \sin(x) dx \end{array} \right.$

$$\begin{aligned} &= -\int (1-u^2) du \\ &= -\left(u - \frac{u^3}{3}\right) + C \\ &= \frac{\cos^3 x}{3} - \cos(x) + C \end{aligned}$$

② (a) $f(x) = \frac{x(x^2+1)^2}{x+2} \Rightarrow \ln |f(x)| = \ln \left(\frac{|x| |x^2+1|^2}{|x+2|} \right)$

$$= \ln|x| + 2\ln|x^2+1| - \ln|x+2|$$

$$\begin{aligned} \Rightarrow \frac{f'(x)}{f(x)} &= (\ln|x|)' + 2(\ln|x^2+1|)' - (\ln|x+2|)' \\ &= \frac{1}{x} + \frac{2 \cdot 2x}{x^2+1} - \frac{1}{x+2} \Rightarrow \quad (\text{next page}) \end{aligned}$$

$$f'(x) = f(x) \left[\frac{1}{x} + \frac{4x}{x^2+1} - \frac{1}{x+2} \right] \quad (2)$$

$$= \frac{x(x^2+1)^2}{x+2} \left[\frac{1}{x} + \frac{4x}{x^2+1} - \frac{1}{x+2} \right] \quad \square$$

This is not defined for $x=0$ or -2 , but multiplying out the

bracket we get

$$f'(x) = \frac{(x^2+1)^2}{x+2} + \frac{4x^2(x^2+1)'}{x+2} - \frac{x(x^2+1)^2}{(x+2)^2}$$

& this is not defined only when $x=-2$.

Finally $f'(1) \square = \frac{1(2^2)}{3} \left[\frac{1}{1} + \frac{4}{2} - \frac{1}{3} \right]$

$$= \frac{32}{9}$$

b) $\int_1^2 x^2 \ln(x) dx = \int_1^2 \frac{(\ln x) x^2 dx}{u \quad dv}$

Let:

$$\frac{dv}{dx} = x^2 \Rightarrow v = \frac{x^3}{3}$$

$$u = \ln x \Rightarrow$$

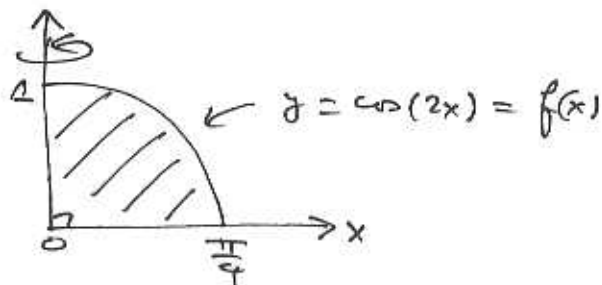
$$\frac{du}{dx} = \frac{1}{x} \Rightarrow du = \frac{dx}{x}$$

$$= uv \Big|_1^2 - \int_1^2 v du$$

$$= \frac{x^3}{3} \ln x \Big|_1^2 - \int_1^2 \frac{x^3}{3} \frac{1}{x} dx$$

$$= \frac{2^3}{3} \ln 2 - 0 - \frac{1}{3} \int_1^2 x^2 dx = \frac{2^3}{3} \ln 2 - \frac{1}{9} x^3 \Big|_1^2$$

$$= \frac{8}{3} \ln 2 - \frac{7}{9}$$



c) Vol = $2\pi \int_0^{\pi/4} x f(x) dx$

$$= 2\pi \int_0^{\pi/4} \frac{x \cos(2x) dx}{u \quad dv}$$

$$= 2\pi \left(uv \Big|_0^{\pi/4} - \int_0^{\pi/4} v du \right)$$

$$= 2\pi \left(\frac{x}{2} \sin(2x) \Big|_0^{\pi/4} - \int_0^{\pi/4} \frac{1}{2} \sin 2x dx \right)$$

$$= 2\pi \left(\frac{\pi}{8} \cdot 1 - 0 - \left(-\frac{1}{4} \cos(2x) \right) \Big|_0^{\pi/4} \right)$$

$$= \frac{\pi^2}{4} + \frac{2\pi}{4} (\cos(\pi/2) - \cos(0)) = \frac{\pi^2}{4} - \frac{\pi}{2} //$$

Let: $u = x \Rightarrow du = dx$

$$\frac{dv}{dx} = \cos(2x) \Rightarrow v = \frac{1}{2} \sin(2x)$$