

MATH101B - Introduction to Calculus

TEST 2

Tuesday 2 October 2007 - 7.05 pm

Time Allowed: 1 hour

Part A - Answer questions on the *ANSWER SHEET* provided.

This is worth 48% of the total marks and you should not spend more than about half the time on it.

Part B - Answer the 2 questions in order. This is worth 52% of the total marks.

No one is to leave the lecture room during the last 15 minutes of the test period.

Calculators (NOT programmable) may be used. Two pages of formulas are available.

PART A

MULTI-CHOICE (each question is worth 4%)

1. $\int_0^1 6(x+5)^4 dx =$

(a) 5^5

(b) $\frac{6^6}{5}$

(c) 1

(d) $6^6 - 5^5$

(e) None of these.

2. Which of the following is an antiderivative for $3e^{-3x} - \sin 2x$?

(a) $\cos(2x) + e^{-3x}$

(b) $\cos(2x) - 9e^{-3x}$

(c) $e^{-3x} - \frac{\cos(2x)}{2}$

(d) $\cos^2(x) - e^{-3x}$

(e) $\frac{\cos(2x)}{2} - \frac{e^{-3x}}{3}$

3. $\int_0^1 (x^2 + 2x)/\sqrt{x} dx =$

(a) $26/15$

(b) $9/7$

(c) $13/15$

(d) $26/5$

(e) None of these.

4. The expression $\sum_{j=1}^n 3\bar{x}_j(1-\bar{x}_j)\Delta x_j$ represents a Riemann sum for the integral of a function $f(x)$.

The function is

(a) $3x - x^3$

(b) $3x(1-x)$

(c) $\int_0^x f(t)dt$

(d) $3x - 3x^2$

(e) None of these.

5. If $F(x) - \sin(x) = \int_0^x (1+t)^2 dt$ then $F'(0) =$

(a) -1

(b) 0

(c) 1

(d) 2

(e) None of these.

6. The length of arc between $x = a$ and $x = b$ of the curve $y = f(x)$, where f and f' are continuous on $[a, b]$, is

(a) $\pi \int_a^b (f(x))^2 dx$

(b) $2\pi \int_a^b x f(x) dx$

(c) $2\pi \int_a^b f(x) \sqrt{1+(f'(x))^2} dx$

(d) $\int_a^b \sqrt{1+(f'(x))^2} dx$

(e) None of these.

7. Which of the following identities is **NOT** correct?

(a) $\cos(2x) = 2\cos^2(x) + 1$

(b) $\cos(2x) = 1 - 2\sin^2 x$

(c) $1 + \tan^2(x) = \sec^2(x)$

(d) $\sin^2(x) + \cos^2(x) = 1$

(e) $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$.

8. Which of the following statements is **NOT** correct?

(a) $y = 2\cos(x)$ satisfies the equation $y'' + y = 0$

(b) $y = 2\sin(x)$ satisfies the equation $y'' + y = 0$

(c) $y = 2\tan(x)$ satisfies the equation $y' = 2(1+y^2)$

(d) $y = 2\ln(x)$ satisfies the equation $y' = 2/x$

(e) $y = 2e^x$ satisfies the equation $y' - y = 0$.

9. If $f: [2, \infty) \rightarrow \mathbb{R}$ is defined by $f(x) = 4 - 4x + x^2$ then $f^{-1}(x)$ is

- (a) undefined (b) $+\sqrt{x} + 2$ (c) $-\sqrt{x} + 2$
(d) $1/(4 - 4x + x^2)$ (e) None of these.

10. If $\int e^{-x} \cos(x) dx = Ae^{-x} \cos x + Be^{-x} \sin x + C$ then

- (a) $A = -1/2$ and $B = 1/2$ (b) $A = -1/2$ and $B = -1/2$ (c) $A = 1/4$ and $B = -1/4$
(d) $A = -1$ and $B = 0$ (e) None of these.

11. The volume of the solid generated when the region under the curve $f(x) = 8x(1-x)$ between $x = 0$ and $x = 1$ is revolved about the line $y = 1$ is given by

- (a) $\int_0^1 \pi f(x)^2 dx$ (b) $\int_0^1 \pi f(x-1)^2 dx$ (c) $\int_0^1 \pi (f(x)-1)^2 dx$
(d) $\int_0^1 \pi (f(x)^2 - 1^2) dx$ (e) None of these.

12. Find the value of the integral $\int_0^{\pi/2} \frac{\cos(x)}{1 + \sin^2(x)} dx$

- (a) 0 (b) $\pi/4$ (c) $\pi/2$
(d) $3\pi/4$ (e) None of these.

PART B

(Each question is worth 26%)

1. Find expressions for the following indefinite integrals

(a) $\int 2x(x^2 + 1)^4 dx$

(b) $\int \frac{x}{\sqrt{1-x^2}} dx$

(c) $\int \frac{y}{\sqrt{y+1}} dy$

(d) $\int \sin^3(x) dx.$

2. (a) Use logarithmic differentiation to find an expression for the derivative of

$$f(x) = \frac{x(x^2 + 1)^2}{(x + 2)}$$

Simplify your expression and note that values of x for which it is well defined.
Compute $f'(1)$.

(b) Evaluate $\int_1^2 x^2 \ln(x) dx.$

- (c) The area under the graph of the function $f(x) = \cos(2x)$ from $x=0$ to $x=\pi/4$ is revolved about the y -axis. Use the method of shells to calculate the volume of the solid of revolution.