

UNIVERSITY OF WAIKATO
Department of Mathematics

MATH101-10B - Introduction to Calculus

TEST 1

Thursday 12 August 2010

Time Allowed: 1 hour and 20 mins

Part A – TEN Short Answer questions worth 5% each (50% of the total marks) - you should not spend more than about half the time on this.

Part B – TWO Long Answer questions worth 25% each (50% of the total marks).

No one is to leave the lecture room **during the last 10 minutes** of the test period.

Calculators may be used but **NO Graphical or Symbolic calculators permitted.**

PART A

SHORT Answer Questions - worth 5% each
(You do not need to show your working)

1. If $f(x) = x^3 + 2x^{1/2} - x^{-1}$ evaluate $f'(4)$.
2. If $g(x) = 2x^3 - 9x^2 + 12x$ find the 2 points where $g'(x) = 0$.
3. Find the equation of the tangent to the curve $y = \frac{1}{x+1}$ at $(0, 1)$.
4. If u, v, w are functions of x find an expression for $\left(\frac{uv}{w}\right)'$ in terms of $u, v, w, u', v',$ and w' .
5. State the chain rule for $(f \circ g)'(a)$ in functional form and then for $\frac{dy}{dx}$ using $y = f(u), u = g(x)$, indicating where the derivatives are evaluated.
6. If $f(x) = \sqrt{9 + (x+1)^4}$ evaluate $f'(1)$.
7. If $f(x) = 5x \sin\left(x^2 + \frac{\pi}{2}\right)$ evaluate $f'(0)$.
8. Find the maximum domain in \mathbb{R} of the function given by the expression $f(x) = \frac{\sqrt{x-4}}{x-5}$.
9. If $f(x) = \begin{cases} x & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$

evaluate $\lim_{x \rightarrow 1} f(x)$.

10. Evaluate $\lim_{x \rightarrow 2} \left(\frac{x^2 + 4}{x + 2}\right) \left(\frac{x^2 - 4}{x - 2}\right)$.

PART B

LONG Answer Questions - worth 25% each
(All working should be given)

1. (a) Let $y = f(x)$ be a function with domain \mathbb{R} and let $a \in \mathbb{R}$. What do we mean by the expressions “ f is differentiable at $x=a$ ” and $f'(a)$?
 - (b) If $f(x) = x^2 + 2x$ use the definition of the derivative as a limit to evaluate $f'(2)$.
 - (c) Assuming the product rule for differentiation, use induction to show $(x^n)' = nx^{n-1}$ for $n = 1, 2, 3 \dots$.
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2. (a) Use implicit differentiation to find y' and y'' at the point $(2, 1)$ on the curve $x^2 - y^2 - x = 1$.
 - (b) Gas is escaping from a spherical balloon at the rate of $2\text{cm}^3/\text{min}$. How fast is the surface area shrinking when the radius is 12cm ?

(You may assume a sphere if radius r has volume $V = \frac{4}{3}\pi r^3$ and surface area $S = 4\pi r^2$.

These formulas will be proved after the break.)