Make up Test 1 math101 — 10B

(1) $f(x) = x^3 + 2x^{-1} \Rightarrow f'(x) = 3x^2 - 2x^{-2}$

so $f'(4) = 3.4^2 - \dfrac{2}{4^2} = \dfrac{3 4^4 - 2}{16} = \boxed{\dfrac{383}{8}} = \boxed{47.875}$

(2) $g(x) = \dfrac{x^3}{3} + \dfrac{x^2}{2} - 6x + 1 \Rightarrow g'(x) = x^2 + x - 6 = (x-2)(x+3)$

$\Rightarrow g'(2) = g'(-3) = 0$ i.e. $\boxed{x \in \{2, -3\}}$

(3) $y = \dfrac{2}{x} \Rightarrow \dfrac{y-2}{x-1} = \left.\dfrac{dy}{dx}\right|_{x=1} = \left.-\dfrac{2}{x^2}\right|_{x=1} = -2$

$\Rightarrow y - 2 = -2x + 2$

$\boxed{y = -2x + 4}$

(4) $(2u + 3uv)' = 2u' + 3(uv)' = 2u' + 3[uv' + u'v] = \boxed{2u' + 3uv' + 3u'v}$

(5) Chain-Rule $(f \circ g)'(a) = f'(g(a)).g'(a)$

OR $u = g(x)$ $\left.\dfrac{dy}{dx}\right|_{x=a} = \left.\dfrac{dy}{du}\right|_{u=g(a)} \cdot \left.\dfrac{du}{dx}\right|_{x=a}$

(6) $f(x) = \sqrt{16 + (x+2)^2} = (16 + (x+2)^2)^{\frac{1}{2}} \Rightarrow$

$\Rightarrow f'(x) = \frac{1}{2}(16 + (x+2)^2)^{-\frac{1}{2}}(0 + 2(x+2)) = \dfrac{x+2}{\sqrt{16 + (x+2)^2}}$

$\Rightarrow f'(1) = \dfrac{3}{\sqrt{4^2 + 3^2}} = \dfrac{3}{\sqrt{5^2}} = \boxed{\dfrac{3}{5}}$

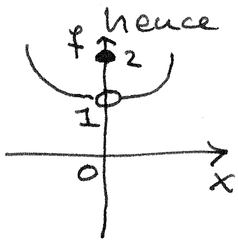
(7) $f(x) = x\cos(x^2) \Rightarrow f'(x) = 1.\cos(x^2) \mp x\,2x.\sin(x^2)$

$\Rightarrow f'(0) = \cos(0^2) - 2 0^2 \sin(0^2) = \cos(0) = \boxed{1}$

(8) $f(x) = \dfrac{\sqrt{x}}{x+3}$ - need $x \geq 0$ for $\sqrt{x}$ to make sense, and
- need. $x \neq -3$ for $\dfrac{1}{x+3}$ to be defined

hence max domain is $\{x \in \mathbb{R} : x \geq 0\} = [0, \infty)$.

(9)



$\lim_{x \to 0} f(x) = 1$ since, when taking the limit, we never have $x = 0$, and we can make $f(x)$ arbitrarily close to 1 when x is sufficiently close to 0 and $x \neq 0$

(10) $\lim_{\theta \to 0} \dfrac{\sin(2\theta)}{\theta}\left(\dfrac{\theta^2+1}{\theta+1}\right) = \lim_{\theta \to 0} 2\dfrac{\sin(2\theta)}{2\theta} \lim_{\theta \to 0}\left(\dfrac{\theta^2+1}{\theta+1}\right) = 2\lim_{\alpha \to 0} \dfrac{\sin(\alpha)}{\alpha}\left(\dfrac{0^2+1}{0+1}\right)$

Let $\alpha = 2\theta$: $= 2.1.1 = 2$ //