A

Marks

1. $\boxed{\dfrac{777}{16}}$ or 48.56 3

2. $\boxed{x = 1 \text{ and } x = 2}$ 3

3. $\dfrac{dy}{dx} = -\dfrac{1}{(x+1)^2} = -1$ @ $x = 0$ 3

$-1 = \dfrac{y-1}{x-0} \Rightarrow \boxed{y = -x + 1}$ 3

4. $\left(\dfrac{uv}{w}\right)' = \dfrac{w(uv)' - uvw'}{w^2} = \boxed{\dfrac{wuv' + wu'v - uvw'}{w^2}}$ 3

5. $(f \circ g)'(a) = f'(g(a)) \cdot g'(a)$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$ where $\dfrac{dy}{du}$ is evaluated at $u = g(a)$

$\dfrac{du}{dx}$ - - - - - $x = a$ 3

$\dfrac{dy}{dx}$ - - - - - - $x = a$

6. $\boxed{\dfrac{16}{5}}$ 3

7. $\boxed{5}$ 3

8. $\boxed{[4, \infty) \setminus \{5\}}$ OR $\{x : 4 \leq x < 5 \text{ or } 5 < x\}$ or $[4,5) \cup (5,\infty)$. 3

9. $\lim\limits_{x \to 1} f(x) = \boxed{1}$ 3

10. $\lim\limits_{x \to 2} = \boxed{8}$ 3

B 1 (a) f is <u>differentiable</u> at $x = a$ means the limit

$\lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$ <u>exists</u> as a real number. When it exists 5

we call its value $f'(a)$, the <u>derivative</u> of $f(x)$ at $x = a$.

(b) $f'(2) = \lim\limits_{h \to 0} \dfrac{f(2+h) - f(2)}{h} = \lim\limits_{h \to 0} \dfrac{(2+h)^2 + 2(2+h) - (2^2 + 2.2)}{h}$

$= \lim\limits_{h \to 0} \dfrac{1}{h}\left[\cancel{4} + \underbrace{\cancel{4h} + \cancel{2h}}_{6h} + h^2 + \cancel{4} - \cancel{4} - \cancel{4}\right] = \lim\limits_{h \to 0}\left(\dfrac{6h + h^2}{h}\right) = \lim\limits_{h \to 0}(6 + h)$ 5

$= 6$ //

1 (c) If $n=1$, $(x^1)' = 1 = 1x^{1-1}$ so the rule is true for $n=1$.

Assume it is true for $n \in \mathbb{N}$ i.e. $(x^n)' = nx^{n-1}$. ⊛

Then $(x^{n+1})' = (x^1 . x^n)' = x.(x^n)' + (x)'x^n$

$$\overset{⊛}{=} x\, n x^{n-1} + 1.x^n$$
$$= nx^n + 1.x^n$$
$$= (n+1)x^n = (n+1)x^{(n+1)-1}$$

So the rule is true for $n+1$. Hence, by induction, it is true for all $n \in \mathbb{N}$. 5

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2 (a) First since $2^2 - 1^2 - 2 = 1$ the point $(2,1)$ is on the curve. Differentiating implicitly $2x - 2y\frac{dy}{dx} - 1 = 0$ ⊠

$$\Rightarrow 2y\frac{dy}{dx} = 2x-1$$
$$\Rightarrow \frac{dy}{dx} = \frac{2x-1}{2y} = y' \quad (\alpha)$$

Diff. ⊠ again $2 - 2(y')^2 - 2yy'' = 0$

$\therefore 1 - (y')^2 - yy'' = 0 \Rightarrow \frac{d^2y}{dx^2} = \frac{1-(y')^2}{y} = y'' \quad (\beta)$ 5

Evaluating (α) at $x=2, y=1 \Rightarrow y' = \frac{2.2-1}{2} = \boxed{\frac{3}{2}}$

Evaluating (β) at $y=1, y'=\frac{3}{2} \Rightarrow y'' = \frac{1-\frac{9}{4}}{1} = \boxed{-\frac{5}{4}}$ 5

(b) $V = \frac{4}{3}\pi r^3$ $\frac{dV}{dt} = -2$

$S = 4\pi r^2$ want $\frac{dS}{dt}$ when $r = 12$.

$-2 = \frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt} = \frac{4}{3}\pi\, 3r^2 \frac{dr}{dt} \Rightarrow \dot{r} = -\frac{1}{2\pi r^2}$

Then $\frac{dS}{dt} = \frac{dS}{dr}\frac{dr}{dt} = 8\pi r\dot{r} = 8\pi r\left(\frac{-1}{2\pi r^2}\right) = -\frac{4}{r}$

Hence at $r = 12$, $\frac{dS}{dt} = -\frac{4}{12} = \boxed{-\frac{1}{3} \text{ cm/min}}$ 5
