

MATH101-08A - Introduction to Calculus

TEST 1

Wednesday 2 April 2008

Time Allowed: 1 hour

Part A - Answer questions on the *ANSWER SHEET* provided.

This is worth 50% of the total marks and you should not spend more than about half the time on it.

Part B - Answer the 2 questions in any order. This is worth 50% of the total marks.

No one is to leave the lecture room during the last 10 minutes of the test period.

Calculators (NOT programmable) may be used.

PART A
MULTI-CHOICE

(Each question is worth 5%. Answer on the sheet provided.)

= correct or best answer

= near answer

1. The domain of the function $f(x) = \frac{\sqrt{x}}{x-2}$ is

(A) \mathbb{R}

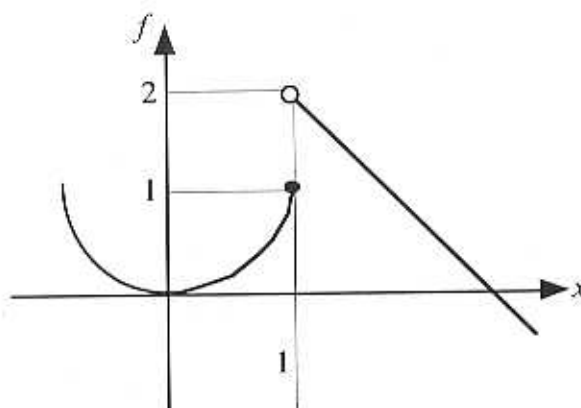
(B) $[0, \infty)$

(C) $\mathbb{R} \setminus \{2\}$

(D) $[0, \infty) \setminus \{2\}$

(E) $(-\infty, 0) \cup \{2\}$.

2. For the function $f(x)$ with the given graph, which of the following statements is NOT correct?



(A) $\lim_{x \rightarrow 1^-} f(x) = 1$

(B) $\lim_{x \rightarrow 1^-} f(x) = f(1)$

(C) $\lim_{x \rightarrow 1^+} f(x) = 2$

(D) f is differentiable at $x = 1$

(E) f is continuous and differentiable at every point except $x = 1$.

3. Let

$$f(x) = \begin{cases} 2 & , \quad x < 1 \\ 1 & , \quad x = 1 \\ x^2 + 1 & , \quad x > 1. \end{cases}$$

Which of the following statements is **NOT** correct?

(A) $\lim_{x \rightarrow 1^+} f(x) = 2$

(B) $\lim_{x \rightarrow 1} f(x)$ does exist

(C) $\lim_{x \rightarrow 1} f(x) = f(1)$

(D) f is **not** continuous on \mathbb{R}

(E) $f'_+(1) = 0$.

4. $\lim_{\theta \rightarrow 0} \frac{\tan(2\theta)}{\theta}$

(A) 2

(B) $\frac{1}{2}$

(C) 1

(D) 0

(E) None of these.

5. The slope of the graph of $y = x^2 + 3x + 7$ at $(-1, 5)$ is

(A) 1

(B) $2x + 3$

(C) 5

(D) $\frac{dy}{dx}$

(E) None of these.

6. The tangent to the curve $y = \frac{x+1}{x-1}$ at $(2, 3)$ has equation

(A) $y = \frac{x+1}{x-1}$

(B) -2

(C) $y = 7 - 2x$

(D) $y = 2x + 7$

(E) None of these.

7. If $f(x) = x^{3/4} - \frac{1}{2}x^{-2}$ then for $x > 0$, $f'(x) =$

(A) $\frac{3}{4}x^{7/4} + x^{-1}$

(B) $\frac{3}{4}x^{-1/4} - x^{-3}$

(C) $\frac{3}{4}x^{1/4} + \frac{1}{x^3}$

(D) $\frac{3}{4x^{1/4}} - \frac{1}{x^3}$

(E) None of these.

8. The slope of the tangent to the curve $4x - xy + y^3 = 7$ at the point $(2, 1)$ is

(A) -3

(B) 3

(C) $\frac{y-4}{3y^2-x}$

(D) Does not exist

(E) None of these.

9. If $f(x) = (2x+3)\sin(x^2+3x+1)$ then $f'(x) =$

(A) $-\cos(x^2+3x+1)$

(B) $\sin(x^2+3x+1)(4x^2+12x+11)$

(C) $2\cos(x^2+3x+1) + (2x+3)^2\sin(x^2+3x+1)$

(D) $-2\cos(x^2+3x+1)$

(E) None of these.

10. The sketched graph is most like the graph of the rational function

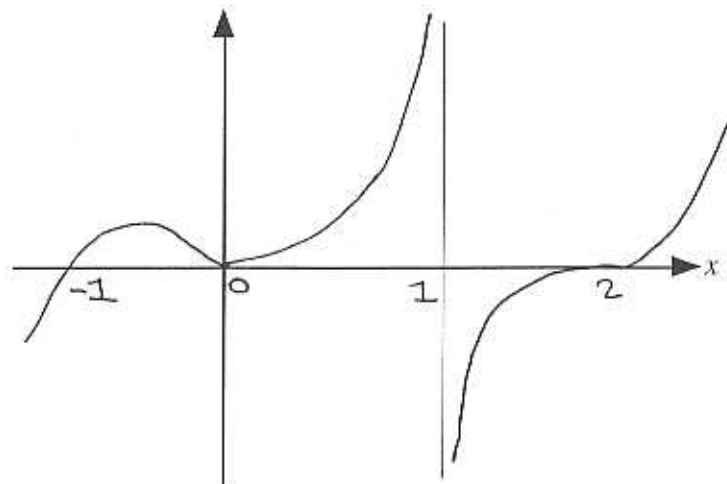
(A) $\frac{(x+1)^2x(x-2)^7}{x-1}$

(B) $\frac{(x+1)(x^2)(x-2)^2}{(x-1)^2}$

(C) $\frac{(x+1)x^2(x-2)^3}{(x-1)^2}$

(D) $\frac{(x+1)(x^2)(x-2)^3}{(x-1)}$

(E) None of these.



PART B

(Each question is worth 25%)

1. (a) Explain briefly what the statements “ f is continuous at $x = a$ ” and “ f is differentiable at $x = a$ ” mean. Give an example of a function which is continuous but not differentiable at $x = 1$.

(b) Let $f(x) = \begin{cases} x^2 & f \quad x \leq 1 \\ ax + b & f \quad x > 1. \end{cases}$

Find values for the constants a and b so that $f(x)$ is continuous and differentiable at $x = 1$.

2. (a) Use the limit definition of the derivative $f'(a)$ to find the derivative of $f(x) = x^3 + 2x^2 + 1$ at $x = a$.

Hint: Use $(a + h)^3 = a^3 + 3a^2h + 3ah^2 + h^3$.

- (b) A girl flies a kite at a height of 300 ft. The wind carries the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?