Part A

PART A

Questions	1	2	3	4	5	6	7	8	9	10	11	12	13
Answers	(d)	(c)	(d)	(e)	(b)	(d)	(b)	(e)	(c)	(b)	(b)	(e)	(d)

Part B

1 (a) By $\lim_{x\to a+} f(x) = L$ we mean we can make $f(x)$ "arbitrarily close to L provided we choose $x > a$ and sufficiently close to a."

By "f is continuous at $x = a$" we mean $\lim_{x\to a} f(x) = f(a)$.

(b)

$$\lim_{x\to -1-} f(x) = \lim_{x\to -1-} (2) = 2$$

$$\lim_{x\to -1+} f(x) = \lim_{x\to -1+} (ax+b) = a(-1)+b = -a+b$$

$$\lim_{x\to 3-} f(x) = \lim_{x\to 3-} (ax+b) = a\,3+b = 3a+b$$

$$\lim_{x\to 3+} f(x) = \lim_{x\to 3+} (-2) = -2$$

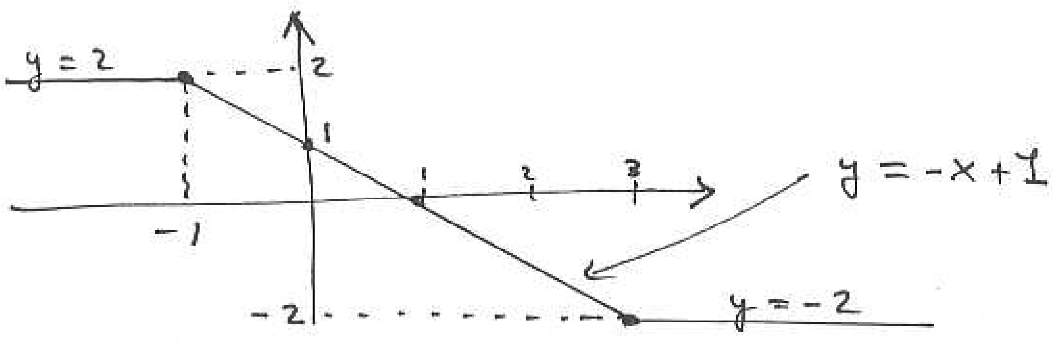
Hence we need $2 = -a+b$ (1) and.

$-2 = 3a+b$ (2)

$\Leftrightarrow\ 2 = -3a-b$ (3)

(1) + (3) $\Rightarrow$ $4 = -4a \Rightarrow a = -1$, then (1) $\Rightarrow$

$b = 2+a$
$= 1$ so $\boxed{a=-1,\ b=1}$



(2) (a) $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$f(x) = \frac{2+x}{x} \Rightarrow$ $\qquad f(1) = 3$

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{2+1+h}{1+h} - \frac{2+1}{1}}{h} \times \frac{1+h}{1+h}$$

$$= \lim_{h\to 0} \frac{3+h - 3(1+h)}{h(1+h)}$$

$$= \lim_{h\to 0} \frac{3+h-3-3h}{h(1+h)} = \lim_{h\to 0} \frac{-2\cancel{h}}{\cancel{h}(1+h)}$$

$$= -2$$

(b) Eqn of the tangent is $-2 = \frac{y-3}{x-1}$

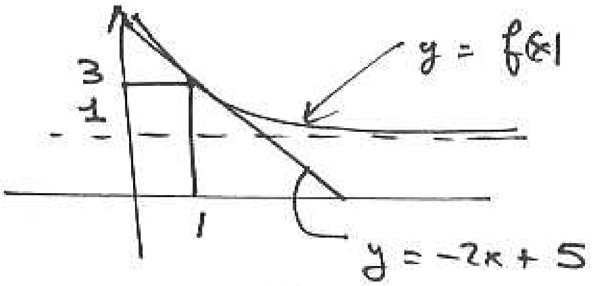
$\Rightarrow \quad y - 3 = -2(x-1)$ or

$\quad y = -2x + 2 + 3 = -2x + 5$

On $(0,\infty)$, $x > 0$ so $f(x) > 0$. $f(x) = \frac{2}{x} + \frac{x}{x} = 1 + \frac{2}{x}$

as $x \to 0+ \quad f(x) \to 1 + \infty = \infty$

as $x \to \infty \quad f(x) \to 1 + 0 = 1$ $\qquad$ Hence



Note that $f'(x) = -\frac{2}{x^2} < 0$ for all $x > 0$, so there are no critical points.

(3) (a) $y = 4(\cos(x))^{-1} + (\tan x)^{-1} \Rightarrow$

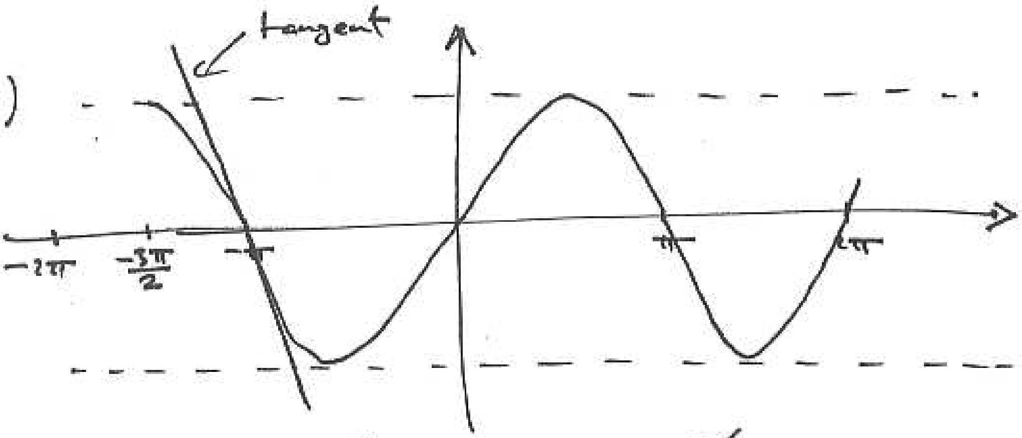
$$\frac{dy}{dx} = -4(\cos(x))^{-2}\cos'(x) - (\tan x)^{-2}\tan'(x)$$

$$= -\frac{4}{\cos^2(x)}(-\sin(x)) - \frac{\sec^2(x)}{\tan^2(x)} = 4\frac{\sin(x)}{\cos^2(x)} - \frac{\sec^2(x)}{\tan^2(x)}$$

But $\tan\left(\frac{\pi}{4}\right) = 1$ and $\sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos \pi/4} = \sqrt{2} \Rightarrow$

at $x = \frac{\pi}{4}$: $\frac{dy}{dx} = 4 \cdot \sqrt{2} - \frac{2}{1} = -2 + 4\sqrt{2}$ //

3 (b)



$f(x) = \sin(x) \Rightarrow f'(x) = \cos(x)$ $\quad f'(-\pi) = \cos(-\pi)$

$= \cos(2\pi - \pi) = \cos(\pi)$

$= -1$

So eqn is $\quad -1 = \dfrac{y - 0}{x - (-\pi)}$ $\quad$ since $\sin(-\pi) = \sin(\pi) = 0$

$\Rightarrow \quad y = -(x+\pi) = -x - \pi$. //

4 (a) $f(x) = x^4 - 8x^2 + 16$ on $[-3, 2]$

$f'(x) = 4x^3 - 16x = 4x(x^2 - 4) = 4x(x-2)(x+2)$

$f''(x) = 12x^2 - 16$

$f'(x) > 0 \Leftrightarrow f(x)$ is increasing. $\quad f(x) = 0 \Leftrightarrow x \in \{-2, 0, 2\}$

$-2 \qquad 0 \qquad 2$

$f'(-3) = -12(-5)(-1) < 0 \longrightarrow$ decreasing on $(-\infty, -2)$

$f'(-1) = 12 > 0 \longrightarrow$ increasing on $(-2, 0)$

$\Rightarrow$ increasing on $(-2, 0)$ and on $(2, \infty)$

$f'(1) = -12 < 0$

$f'(3) = 60 > 0$

(b) Critical points: are $\{-2, 0, 2\}$. Function values

$f(-2) = 0$, $f(0) = 16$, $f(2) = 0$

Singular points: there are none.

End points $\quad f(-3) = 25$, $f(2) = 0$

(c) at $x = -3$, $f'(x) < 0 \Rightarrow$ local maxima,

at $x = -2$ $\quad f''(x) = 32 > 0 \Rightarrow$ local minima,

at $x = 0$ $\quad f''(x) = -16 < 0 \Rightarrow$ local maxima

at $x = 2$ $\quad f''(x) = 32 > 0 \Rightarrow$ local minima

(d) The values of the function are $\{0, 16, 0, 25, 0\}$

Hence the global minimum is 0 at (2 and -2) and global maximum 25 (at -3).