UNIVERSITY OF WAIKATO
Department of Mathematics

MATH101B - Introduction to Calculus

TEST 1

Monday 13 August 2007 - 7.05 pm **Time Allowed: 1 hour 40 mins**

Part A - Answer questions on the *ANSWER SHEET* provided.
This is worth **52%** of the total marks and you should not spend more than about half the time on it.

Part B - Answer the **4 questions** in any order. This is worth **48%** of the total marks.

No one is to leave the lecture room **during the last 15 minutes** of the test period.

Calculators (**NOT** programmable) may be used.

○ = correct or best.

PART A

MULTI-CHOICE (each question is worth 4%)

1. The domain of the function $f(x) = \dfrac{1}{\sqrt{x - x^2}}$ is

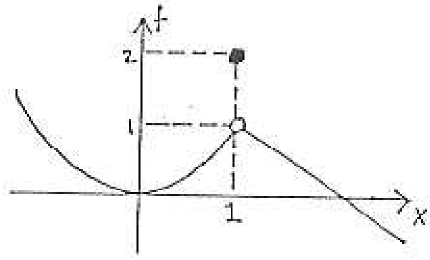
(a) $\mathbb{R}$ (b) $(1, \infty)$ (c) $[0, 1]$

(d) $(0, 1)$ (e) none of these.

2. For the function f with the given graph, which of the following statements is **NOT** correct?

(a) $\lim_{x\to 1+} f(x) = \lim_{x\to 1-} f(x)$ (b) $\lim_{x\to 1} f(x) = 1$ (c) $\lim_{x\to 1} f(x) = 2$

(d) f is not differentiable at $x = 1$ (e) f is continuous at every point except $x = 1$.



3. Let $f(x) = \begin{cases} x & \text{if } x < 1, \\ 2 & \text{if } x = 1, \\ x^2 & \text{if } x > 1. \end{cases}$

Which of the following is **incorrect**?

(a) $\lim_{x\to 1-} f(x) = 1$ (b) $\lim_{x\to 1} f(x) = 1$ does exist (c) $\lim_{x\to 1+} f(x) = \lim_{x\to 1-} f(x)$

(d) f is continuous at $x = 1$ (e) none of these.

4. $\lim_{x\to 1} \dfrac{x^2+x-2}{x^2-1} =$

(a) 2 (b) -2 (c) 1

(d) undefined (e) none of these.

5. The straight line $5x-3y+7=0$ has slope

(a) $-\dfrac{5}{3}$ (b) $\dfrac{5}{3}$ (c) $\dfrac{3}{5}$

(d) $-\dfrac{3}{5}$ (e) $-\dfrac{7}{3}$.

6. The tangent line to the curve $y = 3x + \dfrac{1}{x}$ at the point where $x=1$ has equation

(a) $3 - \dfrac{1}{x^2}$ (b) 2 (c) $y = 3 - \dfrac{1}{x^2}$

(d) $y = 2x+2$ (e) $y = 2$.

7. If $f(x) = \sin(2x)$ then $f'_+(0) =$

(a) 1 (b) 2 (c) $\cos(2x)$

(d) $2\cos(2x)$ (e) none of these.

8. If $f(x) = \dfrac{1}{x^2} + x^{-1/2} + x^3$ then $f'(x) =$

(a) $3x^2 - \dfrac{1}{3x^{3/2}} - \dfrac{2}{x^3}$ (b) $\dfrac{2}{x^3} - \dfrac{1}{2x^{3/2}} + 3x^2$ (c) $3x^{-3} - 2x^{-3/2} + x^2$

(d) $2x^2 - \dfrac{1}{2x^{3/2}} - 2x^{-3}$ (e) none of these.

9. The tangent to the curve $x^2 + xy - y^2 = 1$ at the point $(2, 3)$ has equation

(a) $y = -\frac{4}{7}x + \frac{29}{7}$ (b) $y' = \frac{2x + y}{x - 2y}$ (c) $y = \frac{7}{4}x - \frac{1}{2}$

(d) $y' = \frac{2x}{2y - x}$ (e) none of these.

10. If $f(x) = \left((x+1)^3 - (x-1)^2\right)^2$ then $f'(1) =$

(a) 0 (b) 192 (c) -32

(d) 4 (e) none of these.

11. If $f(u) = u^2 - 3u + 1$ and $u = g(x) = \tan(x)$ then $(fog)'(0) =$

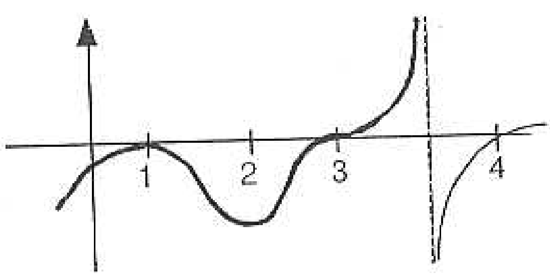
(a) $2u^2 - 3$ (b) -3 (c) -1

(d) 0 (e) none of these.

12. The sketched graph is most like the graph of the rational function:

(a) $\frac{(x-1)^2(x-2)^5(x-4)}{x-3}$ (b) $\frac{(x-1)^2(x-2)^3(x-4)^2}{x-3}$

(c) $\frac{(x-1)^2(x-2)^3(4-x)}{x-3}$ (d) $\frac{(x-1)^3(x-2)^3(x-4)}{3-x}$ (e) none of these.



13. Let $f:[0,2] \to \mathbb{R}$ be defined by $f(x) = x^3 - x$. Then at $\left(\frac{1}{\sqrt{3}}, f\left(\frac{1}{\sqrt{3}}\right)\right)$ f has a

(a) point of inflection (b) local maximum (c) critical point

(d) local minimum (e) none of these.

PART B

(Each question is worth 12%)

1. (a) Explain briefly what $\lim_{x \to a+} f(x) = L$ and "f is continuous at $x = a$" mean.

   (b) Find values for the constants a and b such that the function

$$f(x) = \begin{cases} 2 & x \le -1 \\ ax + b & -1 < x < 3 \\ -2 & x \ge 3 \end{cases}$$

   is continuous and then sketch the graph of $y = f(x)$ on $[-2, 4]$.

2. (a) State the limit definition of the derivative and use it to find the derivative of $f(x) = \dfrac{2+x}{x}$ at $x = 1$.

   (b) Use your answer from (a) to find the equation of the tangent line to $f(x) = \dfrac{2+x}{x}$ at $(1, f(1))$. Sketch the curve and the line on domain $(0, \infty)$.

3. (a) Let $y = \dfrac{4}{\cos x} + \dfrac{1}{\tan x}$.

   Find the value of $\dfrac{dy}{dx}$ when $x = \dfrac{\pi}{4}$.

   (b) Graph the curve of $y = \sin x$ for $-\dfrac{3\pi}{2} \le x \le 2\pi$. Find the equation of its tangent at $x = -\pi$ and include this on your graph.

4. Let $y = f(x) = x^4 - 8x^2 + 16$ with domain $[-3, 2]$.

   (a) Find the intervals on which the function is increasing.

   (b) Find the critical points, singular points and end points, and the corresponding function values.

   (c) Find the functions local maxima and minima.

   (d) Find the global maximum and minimum of the function.