# **Introduction to Calculus** – *Some useful formulas*

### Quadratics

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

### Binomial Theorem

$$(a+b)^n = \binom{n}{0} a^n + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + \binom{n}{n} b^n$$

$$\binom{n}{r} = nC_r = \frac{n!}{(n-r)!r!}$$

### Binomial Expansions

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

### Taylor Expansions

$$\text{for } n \geq 0 : f(a+h) = f(a) + \sum_{j=1}^{n} \frac{f^{(j)}(a)}{j!} h^j + \frac{f^{(n+1)}(\xi)}{(n+1)!} h^{n+1}$$

### Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \operatorname{cos}\operatorname{ec}^2\theta$$

### Products

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

### Compound Angles

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

### Sums

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos C - \cos C = 2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

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**Double Angles**

$$\sin 2A = 2\sin A\cos A$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

**General Solutions**

If $\sin\theta = \sin\alpha$ then $\theta = n\pi + (-1)^n \alpha, n \in \mathbb{Z}$

If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha, n \in \mathbb{Z}$

If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha, n \in \mathbb{Z}$

**Differentiation**

| $y = f(x)$ | $\frac{dy}{dx} = f'(x)$ |
|---|---|
| $c$ | $0$ |
| $x^n$ | $nx^{n-1}, \; n \in \mathbb{R}$ |
| $\ln x$ | $\frac{1}{x}$ |
| $e^{ax}$ | $ae^{ax}$ |
| $a^x$ | $a^x \ln a$ |
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $\tan x$ | $\sec^2 x$ |
| $\sec x$ | $\sec x \tan x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |
| $\cot x$ | $-\operatorname{cosec}^2 x$ |
| $\sin^{-1}\left(\frac{x}{a}\right)$ | $\frac{1}{\sqrt{a^2-x^2}}$ |
| $\tan^{-1}\left(\frac{x}{a}\right)$ | $\frac{a}{a^2+x^2}$ |
| $\ln\lvert\cos x\rvert$ | $-\tan(x)$ |

**Product Rule**

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

**Quotient Rule**

$$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2}$$

**Composite Function Rule**

$$f(g(x))' = f'(g(x)).g'(x)$$

or if $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

**Logarithmic Differentiation**

$$\left(\ln\lvert f(x)\rvert z\right)' = \frac{f'(x)}{f(x)}$$