Paper Code: MATH101-10A(HAM)

THE UNIVERSITY OF
WAIKATO
Te Whare Wānanga o Waikato

2010 A SEMESTER EXAMINATIONS

DEPARTMENT:	Mathematics
PAPER TITLE:	Introduction to Calculus
TIME ALLOWED:	Three Hours
NUMBER OF QUESTIONS IN PAPER:	Section A: SIXTEEN Section B: SIX
NUMBER OF QUESTIONS TO BE ANSWERED:	TWENTY
VALUE OF EACH QUESTION:	Section A: 2.5 marks per question Section B: 15 marks per question
GENERAL INSTRUCTIONS:	Answer ALL questions in SECTION A (worth 40%) and ANY FOUR questions from SECTION B (worth 60%).
SPECIAL INSTRUCTIONS:	NONE
CALCULATORS PERMITTED:	NO

TURN OVER

SECTION A

(Attempt **ALL SIXTEEN** questions - worth 40%)

1. If $f(x) = \sin(g(x))$ and $g(x) = \cos^{-1} x$ determine the composite function $f(g(x))$.

2. Differentiate $y = \tan x + e^{1/x} + x^{1/3}$.

3. Calculate the first and second derivatives of $y = x^2 e^x$.

4. Differentiate $y = e^{\sin\sqrt{x}}$.

5. Differentiate $y = \dfrac{x^2}{x+1}$.

6. Use implicit Differentiation to obtain $\dfrac{dy}{dx}$ in terms of x and y if

$$5y^2 + \sin y = xy + x^2 .$$

7. Determine the derivative of $y = \sin^{-1} x$.

8. Evaluate the $\lim_{x\to -4} \dfrac{2x+8}{(x^2+x-12)}$.

9. Explain why $\lim_{x\to 0} \dfrac{|x|}{x}$ does not exist.

10. Find the most general anti-derivative of $y = f(x) = x^{-5} + xe^{x^2} + \sec^2 x$.

11. Evaluate the integral $\displaystyle\int_0^1 \sqrt{1-x^2}\, dx$.

12. If $\dfrac{1}{x^2+x-2} = \dfrac{A}{x-1} + \dfrac{B}{x+2}$, what are A and B?

13. Find $\displaystyle\int \tan x \sec x\, dx$.

14. Evaluate $\displaystyle\int x\cos x\, dx$.

15. Evaluate $\displaystyle\int xe^{x^2}\, dx$.

16. Verify that $y = 3e^{x^2}$ is a solution of the initial value problem $\dfrac{dy}{dx} = 2xy$ with $y(0) = 3$.

CONTINUED

SECTION B

(Attempt **FOUR** questions - worth a total of 60%)

1. Explain in words (or otherwise) what is meant by the statement

$$\lim_{x \to a} f(x) = L.$$

How can this idea be used to define the continuity of a function $f(x)$ at a point $x = x_0$?

Calculate the derivative of $y = \frac{1}{x} + x$ from the limit definition of the derivative.

Differentiate the following functions:

(i) $\sin^2 x$ (ii) x^x (iii) $x \cos x$.

2. If $g(x)$ and $h(x)$ are differentiable functions establish the quotient rule

$$\frac{d}{dx}\left(\frac{g}{h}\right) = \frac{h\frac{dg}{dx} - g\frac{dh}{dx}}{h^2}.$$

Differentiate the following functions:

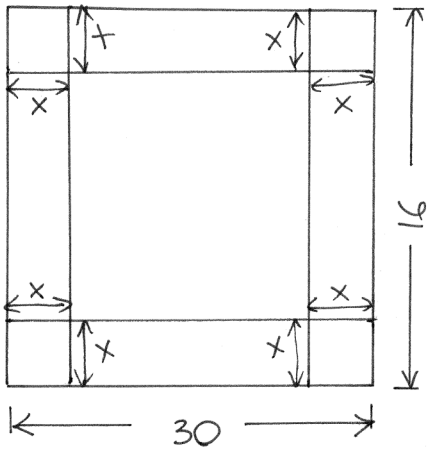
(i) $y = \frac{(x^2 + 1)}{\sin x}$ (ii) $y = \frac{\ell n x}{e^x}$.

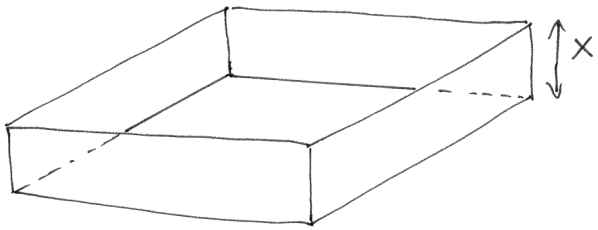
A 5 metre ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 metres/second, how fast will the foot of the ladder be moving away from the wall when the top is 4 metres above the ground?

3. Explain the difference between the absolute maximum and local maxima for a function $f(x)$ defined for $a < x < b$.

   The critical points of a function $f(x)$ are those points for which $f'(x) = 0$. If $f''(x) \neq 0$ at these points what are their nature?

   An open box is to be made from a piece of cardboard 16cm by 30cm by cutting out squares of equal size from the four corners and bending the sides. What size should these squares be in order to obtain a box with largest volume?





4. Establish the integration by parts formula

$$\int u\left(\frac{dv}{dx}\right)dx = uv - \int v\left(\frac{du}{dx}\right)dx$$

   indicating any assumptions you make.

   Hence or otherwise determine the following indefinite integrals:

   (i) $\int x^2 e^x dx$ (ii) $\int \cos^3 x dx$.

   Verify the formula

$$\int \cos^n x dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{(n-1)}{n}\int \cos^{n-2} x dx$$

   by differentiation.

CONTINUED

5. If the natural logarithm ℓnx is defined by

$$\ell nx = \int_1^x \frac{dt}{t}, \qquad x > 0$$

verify the property

$$\ell n(xy) = \ell nx + \ell ny.$$

What is the base of these logarithms?

Is the statement

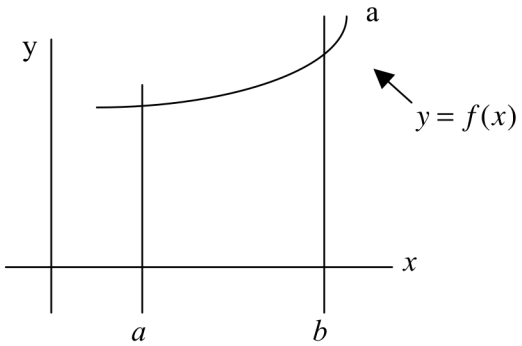
$$\int_1^{1/a} \frac{dt}{t} = -\int_1^a \frac{dt}{t} \qquad , \ a > 0$$

true or false? (Explain).

Evaluate the integrals:

(i) $\int \frac{x}{1+x^2}\,dx$ (ii) $\int \cot x\,dx$.

6. The area under the curve of a positive function $f(x)$ for $a \le x \le b$ is defined by the symbol $\int_a^b f(x)dx$.



If this area is rotated about the x axis show that the corresponding formula for the volume V is

$$\int_a^b \pi f(x)^2 dx .$$

Find the volume obtained by rotating the region bounded by $y = 2 - x^2$, $y = x$ and $x = 0$ about the x axis.