(1) $\mathbb{X} \setminus (A \cup B) = \{x \in \mathbb{X} \mid x \notin A \cup B\}$

$= \{x \in \mathbb{X} \mid x \notin A \text{ and } x \notin B\}$

$= \{x \mid (x \in \mathbb{X} \text{ and } x \notin A) \text{ and } (x \in \mathbb{X} \text{ and } x \notin B)\}$

$= \{x \mid (x \in \mathbb{X} \setminus A) \text{ and } (x \in \mathbb{X} \setminus B)\}$

$= (\mathbb{X} \setminus A) \cap (\mathbb{X} \setminus B)$.

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(2) $\mathbb{X} = \{a, b, c\}$ & $\emptyset \in \tau$. $A_1 = \emptyset$, $A_2 = \{b\}$, $A_3 = \{a, b\}$, $A_4 = \{c, b\}$, $A_5 = \{a, b, c\}$

$\cup$	A_1	A_2	A_3	A_4	A_5
A_1	A_1				
A_2	A_2	A_2			
A_3	A_3	A_3	A_3		
A_4	A_4	A_4	A_5	A_4	
A_5	A_5	A_5	A_5	A_5	A_5.

$\cap$	A_1	A_2	A_3	A_4	A_5
A_1	A_1				
A_2	A_1	A_2			
A_3	A_1	A_2	A_3		
A_4	A_1	A_2	A_2	A_4	
A_5	A_1	A_2	A_3	A_4	A_5.

$\therefore$ τ is closed under $\cup$ and $\cap$ and is finite $\therefore$ it is a <u>topology</u>. It is <u>not</u> Hausdorff: If $c \in P \in \tau$ so does b so we can't separate b and c with open subsets.

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(3) Let $P \subset \mathbb{R}$ be nonempty and open. Then $\exists$ intervals (a_λ, b_λ) with $a_\lambda < b_\lambda$ so $P = \bigcup_{\lambda \in \Lambda} (a_\lambda, b_\lambda)$. If two such intervals meet replace them with their <u>union</u>, so that then $P = \bigcup_{\beta \in B} (a_\beta, b_\beta)$ and the intervals are disjoint. Now each interval contains a rational number, which is unique to that interval once we select it. There are at most a countable number of rational numbers, hence at most a countable number of intervals (could be finite in number).

(4) $d((x,y),(a,b)) := |x-a| + |y-b|$

(M1) $d((x,y)(a,b)) = 0 \iff |x-a| + |y-b| = 0 \iff x = a$ and $y = b$

$\iff (x,y) = (a,b)$.

(M2) $d((x,y),(a,b)) = |x-a| + |y-b| = |a-x| + |b-y|$

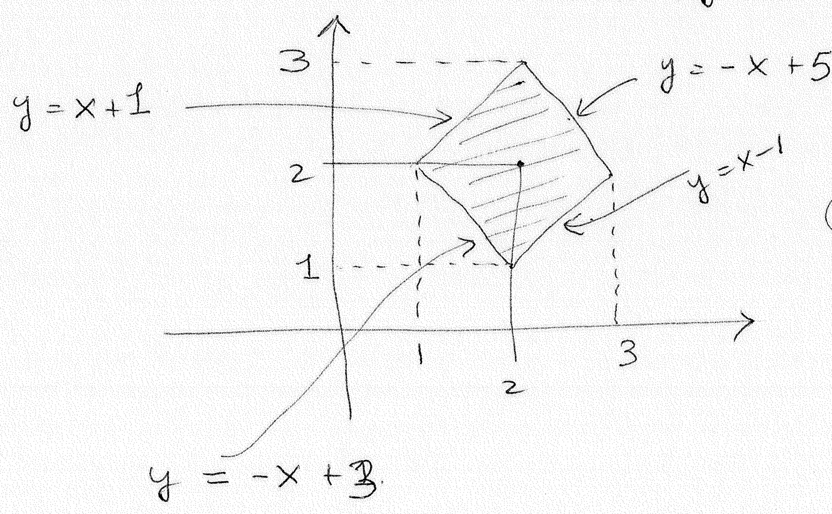
$= (d((a,b),(x,y)))$.

(M3) Given $p_1 = (x_1,y_1)$, $p_2 = (x_2 y_2)$ and $p_3 = (x_3,y_3)$ in $\mathbb{R}^2$

$$\begin{aligned} d(p_1,p_2) &= |x_1 - x_2| + |y_1 - y_2| \\ &= |x_1 - x_3 + x_3 - x_2| + |y_1 - y_3 + y_3 - y_2| \\ &\le |x_1 - x_3| + |x_3 - x_2| + |y_1 - y_3| + |y_3 - y_2| \\ &= |x_1 - x_3| + |y_1 - y_3| + |x_3 - x_2| + |y_3 - y_2| \\ &= d((x_1 y_1),(x_3 y_3)) + d((x_3,y_3),(x_2,y_2)) \\ &= d(p_1,p_3) + d(p_3 p_2) \end{aligned}$$

---

(5) $(x,y) \in B((2,2),1) \iff d((x,y),(2,2)) < 1 \iff$
$|x-2| + |y-2| < 1$



The ball is the interior of the shaded square.

The boundary of the ball occurs when $|x-2| + |y-2| = 1$. To derive this subset of $\mathbb{R}^2$ let. (i) $x \geq 2, y \geq 2$ so $|x-2| + |y-2| =$
$(x-2) + (y-2) = 1$
$\iff y = -x + 5$

(ii) $x < 2$ and $y \geq 2$ etc.

(6) $\underline{\overline{\mathbb{Q}} = \mathbb{R}}$: Let $P \subset \mathbb{R}$ be open and non-empty. Then $\exists$ an interval $(a,b) \subset P$ with $a<b$. Now, by Prop in the notes (a,b) contains a rational number r: $r \in (a,b) \subset P$ and thus $r \in P$. Therefore $\mathbb{Q} \cap P \neq \emptyset$.

Hence $\overline{\mathbb{Q}} = \mathbb{R}$.

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(7) $\underline{A^\circ = A \setminus \partial A}$: Now $A^\circ \subset A$ always by its definition. And since $\partial A = \overline{A} \setminus A^\circ$ no point in ∂A can belong in A°. Hence $A^\circ \subset A \setminus \partial A$. But $A \subset \overline{A}$ always.

$$
\therefore \left\{
\begin{aligned}
A \setminus \partial A &\subset \overline{A} \setminus \partial A \\
&= \overline{A} \setminus (\overline{A} \setminus A^\circ) \\
&= \overline{A} \cap (\overline{A} \setminus A^\circ)^c \\
&= \overline{A} \cap (\overline{A} \cap A^{\circ c})^c \\
&= \overline{A} \cap (\overline{A}^c \cup A^{\circ cc}) \\
&= \overline{A} \cap (\overline{A}^c \cup A^\circ) \\
&= \underbrace{(\overline{A} \cap \overline{A}^c)}_{\emptyset} \cup (\overline{A} \cap A^\circ) \\
&= A^\circ
\end{aligned}
\right.
$$

Hence $A^\circ = A \setminus \partial A$.

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(8) $A \subset X$.

Now. $f^{-1}(B) = \{x \in X : f(x) \in B\}$.

Let $B = f(A)$. so $f^{-1}(f(A)) = \{x \in X : f(x) \in f(A)\}$.

But $a \in A \Rightarrow f(a) \in f(A)$. Hence $A \subset f^{-1}(f(A))$.
