The University of Waikato
Department of Mathematics

Topics in Pure Mathematics: Generating Functions, math319-11A Assignment 1s, due Friday 8th April - through the 252 slot outside G3.19.

1. Find the ordinary power series generating function for each of the following sequences, in simple, closed form as a function of x. In each case the sequence is defined for all integers $n \geq 0$:

$$\begin{aligned} &(a) \qquad a_n = 3n + 2, \\ &(b) \qquad a_n = n^3. \end{aligned}$$

2. Find the ordinary power series generating function for each of the following sequences given by a recurrence relation, in simple, closed form and hence solve the recurrence by finding a formula for a_n as a function of n:

$$\begin{aligned} &(a) \qquad a_{n+2} = 3a_{n+1} - 2a_n, \ a_0 = 0, \ a_1 = 1, \\ &(b) \qquad a_{n+1} = a_n + 3n, \ a_0 = 1. \end{aligned}$$

3. For the Bell numbers $b(n)$, compute $b(4)$ by hand and then, using $B(x) = e^{\epsilon^x - 1}$ and the Series function in Mathematica, derive the expansion of the generating function

$$B(x) = e^{e^x - 1} = 1 + x + x^2 + \frac{5}{6}x^3 + \frac{5}{8}x^4 + O(x^5)$$

so you can check your calculation of $b(4)$.

4. On page 11 of the notes, the derivation of the generating function for the set of legal bracketings using the binomial theorem, was left to the reader. Carry out this derivation and then check the formula for a_n when $n = 3$, first evaluating it by hand and then substituting in the formula.