University of Waikato
Department of Mathematics
Modern Algebra math310-10A Rings and Fields
Assignment 3

Due Friday 21st May 2010

Hand in your work, clearly labelled with your name, id number and the number of the assignment, through the slot "Modern Algebra/310" outside the Mathematics office.

1. Show that $f(x) = x^2 + x + 4$ is irreducible over $\mathbb{Z}_{11}$. Find two different finite fields over which it is reducible and factor $f(x)$ over those fields. Deduce it is irreducible over $\mathbb{Q}$.

2. If R is a ring and $A,\ B \subset R$ are ideals define the product

$$AB := \{\sum_{i=1}^{n} a_i b_i : a_i \in A,\ b_i \in B, n \geq 1\}.$$

Show that AB is also an ideal and that $AB \subset A \cap B$. If $A + B := \{a + b : a \in A,\ b \in B\} = R$ show that $AB = A \cap B$.

3. Let $f(x) = x^3 + x^2 + x + 1 \in \mathbb{Z}_2[x]$. Write $f(x)$ as a product of irreducible polynomials in $\mathbb{Z}_2[x]$.

4. Which of the following polynomials is irreducible over $\mathbb{Q}$?

$$\begin{array}{ll} (a) & x^5 + 9x^4 + 12x^2 + 6, \\ (b) & x^4 + x + 1, \\ (c) & 8x^3 - 6x + 1, \\ (d) & x^5 + 5x^2 + 1. \end{array}$$

5. Show that $R = \mathbb{Z}[\sqrt{-6}]$ is not a unique factorization domain by factoring 10 in two different ways into products. Demonstrate, using the norm or otherwise, that each of the 4 factors is irreducible in R.

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14th May 2010